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E_6 unification model building III. Clebsch-Gordan coefficients in E_6 tensor products of the 27 with higher dimensional representations

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Abstract

E_6 is an attractive group for unification model building. However, the complexity of a rank 6 group makes it non-trivial to write down the structure of higher dimensional operators in an E_6 theory in terms of the states labeled by quantum numbers of the Standard Model gauge group. In this paper, we show the results of our computation of the Clebsch-Gordan coefficients for the products of the **27** with irreducible representations of higher dimensionality: **78**, **351**, **351′**, $\overline{\mathbf{351}}$, and $\overline{\mathbf{351'}}$. Application of these results to E_6 model building involving higher dimensional operators is straightforward.

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1 Introduction

E_6 is the minimal simple gauge group which could accommodate one family of the observed fermions, *and* a family of Higgs states, into a single gauge multiplet.[1] Therefore, unification models based on E_6 can provide relationships for the measured charged fermion masses and quark mixing angles: thirteen unrelated independent parameters of the Standard Model of elementary particles, and at the same time a small set of E_6 symmetric operators may relate the charged fermion data both to the masses and mixings in the neutrino sector and to the parameters of the Higgs sector. In this respect, E_6 provides a framework for the most economic unified supersymmetric theories.

As is well known the key feature among the observed masses of the three generations of fermions is the inter-generational hierarchy. Any unified model has to explain the origin of the hierarchy in terms of the dynamics of the underlying theory. In E_6 models, the hierarchy can follow from the pattern of the symmetry breaking as the rank 6 group is broken down to the Standard Model gauge group, possibly in a succession of steps. The hierarchy may be explicitly realized in terms of higher dimensional operators containing the light states after the superheavy degrees of freedom are integrated out of the effective theory generated below the E_6 breaking scale M_6. [2] From the technical point of view, the construction of higher dimensional E_6 symmetric operators and their structure in terms of the Standard Model states is a non-trivial task. Assuming that the light states occupy the fundamental 27-dimensional irreducible representation (irrep), a complete knowledge of the tensor products of the $\mathbf{27}$ irrep with larger irreps is required. For instance, the E_6 symmetry allows for a higher dimensional operator containing the product of three $\mathbf{27}$s and a $\mathbf{78}$, suppressed by some heavy scale $M_H \geq M_6$. If the $\mathbf{78}$ acquires a vacuum expectation value (vev) $v_6 \approx M_6$ and all three $\mathbf{27}$s contain light states, such an operator contributes to the generation of fermion mass matrices. In particular, for the first two families it may generate entries suppressed by v_6/M_H. Yet, the predictivity of E_6 can only be utilized if the exact form of the singlet in $\mathbf{27} \otimes \mathbf{27} \otimes \mathbf{27} \otimes \mathbf{78}$ in terms of the Standard Model states is known. If two of the $\mathbf{27}$s are contracted antisymmetrically, one needs to know the Clebsch-Gordan decomposition of the $\mathbf{351}$ in the tensor product $\mathbf{27} \otimes \mathbf{78}$, then the decomposition of the $\overline{\mathbf{27}}$ in the product $\mathbf{27} \otimes \mathbf{351}$, and, lastly, the decomposition of the singlet in the product $\mathbf{27} \otimes \overline{\mathbf{27}}$. However, a complete information on general tensor products of exceptional group E_6 is difficult to obtain. [3] As for particular computations, to our knowledge only the Clebsch-Gordan decompositions of $\mathbf{27} \otimes \mathbf{27}$, $\mathbf{27} \otimes \overline{\mathbf{27}}$, and $\mathbf{78} \otimes \mathbf{78}$ are presently available in the literature. [4, 5, 6]. (We note in passing that a separate paper [7] obtains a subset of the results needed for the operator $\mathbf{27^3 78}$ by studying a branching chain of E_6. The results presented in this paper are relevant for the case when the vev is acquired by a zero weight state of the $\mathbf{78}$.)

In this paper, we continue in our earlier work [5, 6] and provide basic group-theoretical tools for a construction of higher-dimensional E_6 symmetric operators. In particular, we present the results of our computation of the Clebsch-Gordan coefficients (CGCs) for the tensor products involving 27, 78, and 351-dimensional representations, the lowest dimensional irreps in E_6. Section 2 contains some necessary mathematical background for our study, mostly concerned with lowering in the weight system of these irreps. Our main results can be found in section 3, where we also comment on the construction and properties of the weight systems of the resulting irreps. Section 4 contains

the summary, while one appendix provides details on the lowering relations in the presence of degenerate weights.

2 Mathematical Preliminaries

In this work we consider tensor products of the fundamental 27-dimensional irrep with higher dimensional 78 and 351-dimensional representations of E_6. In particular, we compute the Clebsch-Gordan coefficients for the products

$$\mathbf{27}\otimes \mathbf{78} = \mathbf{1728}\oplus \mathbf{351}\oplus \mathbf{27}, \tag{1a}$$
$$\mathbf{27}\otimes \mathbf{351} = \overline{\mathbf{7371}}\oplus \overline{\mathbf{1728}}\oplus \overline{\mathbf{351}}\oplus \overline{\mathbf{27}}, \tag{1b}$$
$$\mathbf{27}\otimes \mathbf{351'} = \overline{\mathbf{7722}}\oplus \overline{\mathbf{1728}}\oplus \overline{\mathbf{27}}, \tag{1c}$$
$$\mathbf{27}\otimes \overline{\mathbf{351}} = \overline{\mathbf{5824}}\oplus \mathbf{2925}\oplus \mathbf{650}\oplus \mathbf{78}, \tag{1d}$$
$$\mathbf{27}\otimes \overline{\mathbf{351'}} = \mathbf{3003}\oplus \overline{\mathbf{5824}}\oplus \mathbf{650}. \tag{1e}$$

Before we discuss the construction of the weight systems of the irreps on the right hand side of these relations let us start first with the rules for the construction of the irreps on the left.

The key ingredient of our procedure is the lowering operation which is used to construct a complete weight system by successive application of generators $E_{-\alpha_1}, \ldots E_{-\alpha_6}$. These are the generators which lie outside of the diagonal Cartan subalgebra and correspond to the six simple roots of E_6. (Our choice of generators is described in more detail in [6], and basically follows the standard conventions of [8].) The six generators act as ladder operators — at each level the weight of the new state is obtained from the weight at the previous level by subtracting (in the weight space) the respective simple root:

$$E_{-\alpha_i}|\,w\rangle \;= N_{-\alpha_i,\,w}|\,w-\alpha_i\rangle. \tag{2}$$

For the weight systems of the **27** and **78** constants $N_{-\alpha_i,w}$ satisfy (see eq.(12) in [6])

$$|N_{-\alpha_i,(w)_j}|^2 = \langle\alpha_i|\,w\,\rangle \;+\; |\,\langle(w)_j\,|\,(w)_i\rangle|^2 \;\; |N_{-\alpha_i,\,w+\alpha_i}|^2. \tag{3}$$

It is understood that the new state $|\,w-\alpha_i\rangle$ does not exist if $N_{-\alpha_i,w}=0$ or the r.h.s. of (3) turns out to be negative. The subscript on the weight (w) is only relevant for the six degenerate zero weight states of the **78** and is to be ignored for non-degenerate weights. In fact, the second term on the r.h.s. of (3) never contributes when one constructs the weight system of the **27** as there are no higher multiplets than doublets for any $SU(2)$ subgroup.

We remark that throughout this work, and consistent with our previous studies [5, 6], the lowering phase convention which always fixes constants N to be real and non-negative

$$N_{-\alpha_i,\,w}\geq 0 \tag{4}$$

is adopted for any simple root α_i and any weight system. Then for the zero weight states of the **78** the inner product in (3) can be expressed as

$$\langle(w)_i\,|\,(w)_j\rangle = |A_{ij}|/2, \tag{5}$$

where $A_{ij} \equiv \langle \alpha_i \,|\, \alpha_j \rangle$ are the elements of the Cartan matrix of E_6. [8, 6] This result follows from the decomposition of the **78** weight states into the states of the fundamental representations in the tensor product $\mathbf{27} \otimes \overline{\mathbf{27}}$. [5]

In the appendix, we derive a generalized relation for $N_{-\alpha_i,w}$ for a weight system with multiple degenerate weights at different levels. We now discuss how to apply general formula (23) to the weight systems of the 351-dimensional representations which appear on the left in eq.(1b–e). These irreps, although already rather large, are still special because for each weight subspace a basis can be defined such that the application of a lowering ladder operator results in a single basis state, as indicated in eq.(2). (For larger irreps, there are lowerings which lead to a linear combination of the basis states regardless of the basis definition.) Moreover, if weight (w) is degenerate and a state with weight $(w-\alpha)$ exists, then the $(w+\alpha)$ weight state is either non-degenerate or does not exist at all. Thus for the weight system of the $\mathbf{351}'$ or $\mathbf{351}$ eq.(23) reduces to a simple form

$$|N_{-\alpha,\,(w)_a \to (w-\alpha)_{A_a}}|^2 = \langle \alpha|\, w \rangle \;+\; |\langle (w)_a \,|\, (w)_c \rangle|^2 \;\; |N_{-\alpha,\,(w+\alpha)\to(w)_c}|^2, \tag{6}$$

where, formally, the summation over c is assumed in the last term, but no more than one state actually contributes. Concrete applications of this formula are provided at the end of the section.

Compared to eq.(3) both weights (w) and $(w-\alpha)$ can now be degenerate. We find, however, that in the $\mathbf{351}'$ or $\mathbf{351}$ the $(w+\alpha)$ weight state does not exist if $(w-\alpha)$ is a degenerate weight. Hence if both (w) and $(w-\alpha)$ are degenerate, A_a can be set to a by definition and (6) can be simplified even further:

$$|N_{-\alpha,\,(w)_a \to (w-\alpha)_a}|^2 = \langle \alpha|\, w \rangle, \qquad \text{for both } \;(w)\; \text{ and } \;(w-\alpha)\; \text{ degenerate.} \tag{7}$$

This shows that the definition of the basis states (and their subscript labeling) in the degenerate subspaces of the $\mathbf{351}'$ or $\mathbf{351}$ can be induced from the basis states at the previous level. However, once (w) is found degenerate, how do we know if $(w-\alpha)$ is going to be degenerate and what the dimensionality of this subspace is going to be? Similarly to the case of the **78** weight system [6] the decomposition of the 351-dimensional irreps into the states of the fundamental representations can be recalled. The product $\overline{\mathbf{27}} \otimes \overline{\mathbf{27}} = \mathbf{351}' \oplus \mathbf{351} \oplus \mathbf{27}$ is conjugated to the product studied in ref.[4]. We refer to this work to claim that all degenerate weight subspaces in the $\mathbf{351}'$ (or $\mathbf{351}$) are of the same dimensionality and that the degenerate weights follow the weight system of the **27**. In the end it thus turns out that complete bases in the degenerate weight subspaces of the $\mathbf{351}'$ can be obtained starting from the four (100000) weight states at level 8 of the $\mathbf{351}'$ [9] :

$$\begin{array}{rcl} |\,(100000)_3\rangle &=& E_{-\alpha_3}\,|\,(1\bar{1}2\bar{1}0\bar{1})\rangle\,/\sqrt{2}, \\ |\,(100000)_4\rangle &=& E_{-\alpha_4}\,|\,(10\bar{1}2\bar{1}0)\rangle\,/\sqrt{2}, \\ |\,(100000)_5\rangle &=& E_{-\alpha_5}\,|\,(100\bar{1}20)\rangle\,/\sqrt{2}, \\ |\,(100000)_6\rangle &=& E_{-\alpha_6}\,|\,(10\bar{1}002)\rangle\,/\sqrt{2}. \end{array} \tag{8}$$

With lowering convention (4) the remaining 26 degenerate weight subspaces at lower levels can be specified as $|\,(w)_a\rangle = E_{-\alpha_A} \ldots E_{-\alpha_B}\,|\,(100000)_a\rangle$, $a = 3,4,5,6$, where $E_{-\alpha_A} \ldots E_{-\alpha_B}$ is the lowering

path leading to state $|(w)\rangle$ in the **27**. For the **351** the only difference is that the degenerate weight subspaces are five-dimensional and the relations analogous to (8) also include

$$|\,(100000)_2\rangle = E_{-\alpha_2}\,|\,(02\bar{1}000)\rangle\,/\sqrt{2} \tag{9}$$

when computing the (100000) states at level 7 of this irrep. We note that with this notation the inner product in any degenerate weight subspace of both the $\mathbf{351'}$ and **351** satisfies

$$\langle (w)_a\,|\,(w)_b\rangle = |A_{ab}|/2 \tag{10}$$

(where $a, b = 3, 4, 5, 6$ for the $\mathbf{351'}$, and $a, b = 2, 3, 4, 5, 6$ for the **351**), in a close similarity to the degenerate zero weight subspace of the **78**, eq.(5).

As an example of the application of formula (6) consider all possible lowerings of the state $|\,F_5\rangle$ in the $\mathbf{351'}$ where, for brevity, F stands for the (100000) weight. Three different states at the next level can be obtained: $E_{-\alpha_1}\,|\,F_5\rangle = N_1\,|\,(\bar{1}10000)_5\rangle$, $E_{-\alpha_4}\,|\,F_5\rangle = N_4\,|\,101\bar{2}10\rangle$, and $E_{-\alpha_5}\,|\,F_5\rangle = N_5\,|\,1001\bar{2}0\rangle$. Based on (6) the constants are

$$\begin{array}{rcccccl} |N_1|^2 &=& 1 &+& 0 &=& 1\,, \\ |N_4|^2 &=& 0 &+& (\frac{1}{2})^2(\sqrt{2})^2 &=& \frac{1}{2}\,, \\ |N_5|^2 &=& 0 &+& 1^2\,(\sqrt{2})^2 &=& 2\,, \end{array}$$

as we have already showed in the second and third equation (8) that

$$N_{-\alpha_4,\,(F+\alpha_4)\to(F)_4} \;=\; N_{-\alpha_5,\,(F+\alpha_5)\to(F)_5} \;=\; \sqrt{2}.$$

Implicitly, we also used the fact that (8) represents the only way how the (100000) weight states can be obtained from the states at the previous level. Note that (7) could be used to compute N_1 since both (100000) and $(\bar{1}10000)$ are degenerate weights.

Finally, we remark that the properties of the $\overline{\mathbf{351'}}$ and $\overline{\mathbf{351}}$ are easily derived from the properties of the $\mathbf{351'}$ and **351** after the Dynkin coordinates [and any other indices in Dynkin formalism, like e.g., the labeling of states in eq.(8)] 1 and 2 are exchanged with 5 and 4, respectively.

3 Construction of Clebsch-Gordan Coefficients

Tensor products in eq.(1) can be expressed in terms of the highest weights as

$$(100000)\otimes(000001) \;=\; (100001)\oplus(000100)\oplus(100000), \tag{11a}$$
$$(100000)\otimes(000100) \;=\; (100100)\oplus(000011)\oplus(010000)\oplus(000010), \tag{11b}$$
$$(100000)\otimes(000020) \;=\; (100020)\oplus(000011)\oplus(000010), \tag{11c}$$
$$(100000)\otimes(010000) \;=\; (110000)\oplus(001000)\oplus(100010)\oplus(000001), \tag{11d}$$
$$(100000)\otimes(200000) \;=\; (300000)\oplus(110000)\oplus(100010). \tag{11e}$$

For each product we start with the construction of the weight system of the first irrep on the r.h.s..

The highest weight state of this irrep is non-degenerate and can always be expressed as a trivial combination of the highest weight states of the two irreps on the left-hand side, with the CGC being equal to +1. In the absence of a simple method how to determine the bases in the degenerate weight subspaces which follow at lower levels for each of these irreps, we compute directly the complete weight system in each case. However, note that simple lowering (2) does not necessarily hold for weights with multiple degeneracies, as is discussed in the appendix. States at lower levels are then computed by successive lowerings applied to the states of the **27**, and **78** in case (a) or one of the 351-dimensional irreps in cases (b)–(e). These lowerings were described in detail in the previous section. The computed state is accepted and kept as a new basis state if it cannot be expressed as a linear combination of the previously obtained basis states with the same weight.

It is not necessary to show the Clebsch-Gordan coefficients for every linearly independent state, since there are many states with the same CGCs. Instead, we present the results just for the dominant weight states. Dominant weights are weights with all Dynkin coordinates non-negative. The CGCs for the remaining states can then be determined using the charge conjugation operators [10, 11], or in a straightforward way by direct lowering. In tables 1–5 we present lowering paths for the dominant weight states of the **1728**, $\overline{\mathbf{7371}}$, $\overline{\mathbf{7722}}$, $\overline{\mathbf{5824}}$, and **3003** irreps. In our abbreviated notation, lowering path, let's say, 3421 stands for $E_{-\alpha_3}E_{-\alpha_4}E_{-\alpha_2}E_{-\alpha_1}$ applied (from the right) to the highest weight state. The lowering paths in tables 1–5 actually specify our choice of bases for particular dominant weight subspaces. Explicit Clebsch-Gordan decomposition of the dominant weight states is important because, typically, the multiplicity of degeneracy (i.e., the dimensionality of the weight subspace) changes compared to the degeneracy at the previous level. Clearly, that is why these states cannot be obtained by generalized charge conjugation from the states at the previous levels. Moreover, it is important to check the completeness of a reducible dominant weight subspace. If it is impossible to complete its basis by lowering the states at the previous level, new weight systems open up and the remaining basis vectors are their highest weight states. This is what happens for every dominant weight in the tensor products $\mathbf{78}\otimes\mathbf{78}$ [6], $\mathbf{27}\otimes\overline{\mathbf{27}}$ [5], or $\mathbf{27}\otimes\mathbf{27}$ [5] studied in the earlier work. However, this property of the dominant weights is no longer true for the products studied here. We now discuss shortly the dominant weights in each of the products in (11).

(a) ***(100000)*** $\otimes$ ***(000001)*** $=$ ***(100001)*** $\oplus$ ***(000100)*** $\oplus$ ***(100000)***

At level 4 of the 1728-dimensional **(100001)** irrep we find four states with weight (000100). This weight space, however, is five-dimensional, and the computation of the state orthogonal to the previous four yields the highest weight state of the **351** irrep. (See table 6.) Similarly, at level 11 we find 16-fold degenerate weight (100000), while this reducible subspace unfolds to be 22-dimensional. Since there are five distinct states of the same weight in the **351**, there is room for one extra state. Once computed as orthogonal to all the other 21 states it becomes the highest weight state of the fundamental 27-dimensional **(100000)** irrep. Note that in table 7 we keep the labeling of the five (100000) states of the **351** consistent with the notation introduced in equations (8) and (9).

(b) ***(100000)*** $\otimes$ ***(000100)*** $=$ ***(100100)*** $\oplus$ ***(000011)*** $\oplus$ ***(010000)*** $\oplus$ ***(000010)***

Lowering down to level 4 of the 7371-dimensional **(100100)** irrep we obtain four distinct (000011) weight states spanned over a five-dimensional reducible subspace. The last basis state in this subspace, orthogonal to the four from the $\overline{\mathbf{7371}}$, becomes the highest weight state of the $\overline{\mathbf{1728}}$. (See table 8.) This is a conjugate irrep to the **1728** described in *(a)*. The lowering paths to the dominant weights in its weight system can be obtained from table 1 (replacing 1 and 2 with 5 and 4, and *vice versa*. Proceeding to level 7 a five-fold degenerate (200000) dominant weight is found:

$$|200000_a\rangle = |100000\rangle\,|100000_a\rangle, \qquad a = 2, \dots 6 \tag{12}$$

which, obviously, does not leave any extra space for states outside of the $\overline{\mathbf{7371}}$. This is consistent with no observation in *(a)* of a dominant weight (000020) in the weight system of the **1728**, and also with the fact that there is no (**200000**) irrep on the right side of (11b). The charge conjugation operators can be used to show that a five-fold degenerate weight subspace with CGCs equal to 1 is then present at odd levels of the $\overline{\mathbf{7371}}$ from this level down until subspace $(000\overline{2}0)$ emerges at level 39. Next, at level 8 fifteen linearly independent (010000) weight states are present, while the weight subspace turns out to be 20-dimensional. Not surprisingly there are four states which belong to the weight system of the $\overline{\mathbf{1728}}$ (compare with *(a)* above), and the remaining basis state, orthogonal to the previous nineteen, represents the highest weight state of the $\overline{\mathbf{351}}$. Lastly, at level 15 we get the reducible (000010) weight subspace, which is 66-dimensional. That makes room for the highest weight of the $\overline{\mathbf{27}}$, since there are 44 basis states present in the $\overline{\mathbf{7371}}$ together with 16 states of the $\overline{\mathbf{1728}}$. Additional five states of the $\overline{\mathbf{351}}$ should be expected based on eqs.(8, 9). The CGCs for this subspace are presented in tables 10 and 11.

(c) ***(100000)*** $\otimes$ ***(000020)*** $=$ ***(100020)*** $\oplus$ ***(000011)*** $\oplus$ ***(000010)***

In the construction of the 7722-dimensional (**100020**) irrep one finds a dominant weight already at level 1,

$$|100100\rangle = |100000\rangle\,|000100\rangle. \tag{13}$$

It occupies a one-dimensional subspace, which is consistent with the absence of the (**100100**) irrep in product (11c). The first degenerate dominant weight is obtained at level 5. There, a six-dimensional (000011) weight subspace contains five linearly independent states of the $\overline{\mathbf{7722}}$. The basis in this reducible subspace is completed by the highest weight state of the $\overline{\mathbf{1728}}$ (table 12). Proceeding further, there is no room for the highest weight state of a new irrep when dominant weights (200000) and (010000) are encountered at levels 8 and 9, respectively. The (200000) subspace is four-dimensional and its basis can be specified as in eq.(12). (The states are now numbered as $a = 3, 4, 5, 6$.) The CGC decomposition of the (010000) subspace can be found in table 13. Finally, at level 16 the last dominant weight in this product is unveiled. The reducible (000010) weight subspace turns out to be 57-dimensional, with 40 basis states coming from the $\overline{\mathbf{7722}}$ and 16 states from the $\overline{\mathbf{1728}}$. The remaining state, orthogonal to them, becomes the highest weight state of the $\overline{\mathbf{27}}$ (see tables 14 and 15).

(d) ***(100000)*** $\otimes$ ***(010000)*** $=$ ***(110000)*** $\oplus$ ***(001000)*** $\oplus$ ***(100010)*** $\oplus$ ***(000001)***

In this product, we find the dominant weight states encountered already in the decomposition of $\mathbf{78} \otimes \mathbf{78}$ and $\mathbf{27} \otimes \overline{\mathbf{27}}$. At level 2 of the (**110000**) weight system (*i.e.*, the $\overline{\mathbf{5824}}$ irrep) we reach the

3-dimensional (001000) subspace, with two states in the $\overline{\mathbf{5824}}$ and the third one being the highest weight state of the **2925**, as shown in table 16. Then following the lowering paths in table 4, table II in Ref.[6], and table I in Ref.[5] the dominant weights (100010), (000001), and (000000) follow at levels 7, 12, and 23, respectively. The CGCs for these dominant weights can be found in tables 17–23. The reducible (000000) subspace is 135-dimensional and represents the most (technically) involved computation in this study. Obviously, it cannot (and does not) leave any room for the singlet since the two representations in the product are not conjugate to each other.

(e) ***(100000)*** $\otimes$ ***(200000)*** $=$ ***(300000)*** $\oplus$ ***(110000)*** $\oplus$ ***(100010)***

The 3003-dimensional (**300000**) irrep contains a dominant weight already at level 1:

$$|110000\rangle = (\,|\bar{1}10000\rangle\,|200000\rangle + \sqrt{2}\,|100000\rangle\,|010000\rangle)/\sqrt{3}. \tag{14}$$

The orthogonal combination

$$|110000\rangle = (\,\sqrt{2}|\bar{1}10000\rangle\,|200000\rangle - |100000\rangle\,|010000\rangle)/\sqrt{3} \tag{15}$$

forms the highest weight state of the $\overline{\mathbf{5824}}$. Since then, the same dominant weights occur as in the weight system of the $\overline{\mathbf{5824}}$ described under *(d)* above. There are, however, no **2925** and **78** irreps in this product (see tables 24, 26, and 27), just the highest weight state of the **650** completes the 13-dimensional (100010) subspace at level 8 (table 25). The reducible (000000) weight subspace is 108-dimensional and its decomposition can be found in tables 28–31.

4 Summary

We have presented the Clebsch-Gordan decomposition of the E_6 tensor products of the fundamental **27** irrep with the 78- and 351-dimensional irreps. Analogous products involving the $\overline{\mathbf{27}}$ instead of the **27** can now be obtained trivially by charge conjugation. It is straightforward to apply these results to the construction of higher dimension operators in E_6 model building [12].

Appendix: The problem of Degenerate Weights

Rules (2,3) are insufficient for representations with degenerate weights at successive levels. For degenerate weights we must first identify a particular basis. Label the degenerate basis states of weights $(w+\alpha)$, (w), and $(w-\alpha)$ as

$$\begin{array}{llll} |\;(w+\alpha)_\Gamma\rangle, & \text{where} & \Gamma = 1,\dots D_{w+\alpha}, & \\ |\quad w_c \qquad\rangle, & \text{where} & c = 1,\dots D_w, & \text{and} \\ |\;(w-\alpha)_C\rangle, & \text{where} & C = 1,\dots D_{w-\alpha}. & \end{array}$$

D_w stands for the degeneracy of (w). The basis states are in general non-orthogonal. In our notation, they are always normalized to unity: $\langle w_c \mid w_c\rangle = 1$. The identity operator in the

degenerate subspace is

$$\begin{aligned} I &= G_{ab} \mid w_a \rangle \langle w_b \mid, \\ G_{ab} &= (M^{-1})_{ab}, \quad \text{where} \quad M_{ab} = \langle w_a \mid w_b \rangle. \end{aligned} \tag{16}$$

Although the basis is non-orthogonal, we can construct state vectors which are orthogonal to any state except the state we are interested in

$$\begin{aligned} \mid \hat{w}_b \rangle &= \mid w_a \rangle G_{ab}, \\ \langle \hat{w}_a \mid &= G_{ab} \langle w_b \mid, \end{aligned} \tag{17}$$

which satisfy

$$\langle w_c \mid \hat{w}_a \rangle = \langle \hat{w}_a \mid w_c \rangle = \delta_{ac}. \tag{18}$$

A general raising or lowering of a degenerate weight state can be written as:

$$E_{\alpha_i} \mid w_c \rangle = N_{\alpha_i, w_c \to (w+\alpha_i)_\Gamma} \mid (w+\alpha_i)_\Gamma \rangle \tag{19}$$

$$E_{-\alpha_i} \mid w_c \rangle = N_{-\alpha_i, w_c \to (w-\alpha_i)_C} \mid (w-\alpha_i)_C \rangle \tag{20}$$

where there is a possible sum over the states on the right hand side [compare (20) with (2)]. The lowering normalization constant can then be expressed only as a sum of matrix elements $N_{-\alpha_i, w_a \to (w-\alpha_i)_A} = G^{(w-\alpha_i)}_{AB} \langle (w-\alpha_i)_B \mid E_{-\alpha_i} \mid w_a \rangle$.[13] Using $E_\alpha = E^\dagger_{-\alpha}$ and the defining relation (20) we derive

$$\begin{aligned} & N_{-\alpha, w_a \to (w-\alpha)_A} N^*_{-\alpha, w_b \to (w-\alpha)_B} \langle (w-\alpha)_B \mid (w-\alpha)_A \rangle \\ &\quad = \langle w_b \mid E_\alpha E_{-\alpha} \mid w_a \rangle \\ &\quad = \langle w_b \mid [E_\alpha, E_{-\alpha}] + E_{-\alpha} E_\alpha \mid w_a \rangle \\ &\quad = \langle w_b \mid w_a \rangle \langle \alpha, w \rangle + G^{w+\alpha}_{\Gamma\Delta} \langle w_b \mid E_{-\alpha} \mid (w+\alpha)_\Gamma \rangle \langle (w+\alpha)_\Delta \mid E_\alpha \mid w_a \rangle \\ &\quad = \langle w_b \mid w_a \rangle \langle \alpha, w \rangle + G^{w+\alpha}_{\Gamma\Delta} N_{-\alpha, (w+\alpha)_\Gamma \to w_c} \langle w_b \mid w_c \rangle N^*_{-\alpha, (w+\alpha)_\Delta \to w_d} \langle w_d \mid w_a \rangle. \end{aligned}$$

Hence

$$\begin{aligned} & (G^{w-\alpha})^{-1}_{AB} N_{-\alpha, w_a \to (w-\alpha)_A} N^*_{-\alpha, w_b \to (w-\alpha)_B} \\ &\quad = (G^w)^{-1}_{ab} \langle \alpha, w \rangle + G^{w+\alpha}_{\Gamma\Delta} (G^w)^{-1}_{bc} (G^w)^{-1}_{da} N_{-\alpha, (w+\alpha)_\Gamma \to w_c} N^*_{-\alpha, (w+\alpha)_\Delta \to w_d}. \end{aligned} \tag{22}$$

For $a = b$ we get:

$$\begin{aligned} & (G^{w-\alpha})^{-1}_{AB} N_{-\alpha, w_a \to (w-\alpha)_A} N^*_{-\alpha, w_a \to (w-\alpha)_B} \\ &\quad = \langle \alpha, w \rangle + G^{w+\alpha}_{\Gamma\Delta} (G^w)^{-1}_{ac} (G^w)^{-1}_{ad} N_{-\alpha, (w+\alpha)_\Gamma \to w_c} N^*_{-\alpha, (w+\alpha)_\Delta \to w_d}. \end{aligned} \tag{23}$$

Another useful expression can be found by contracting relation (22) with G^w_{ab}:

$$\begin{aligned} & (G^w)_{ab} (G^{w-\alpha})^{-1}_{AB} N_{-\alpha, w_a \to (w-\alpha)_A} N^*_{-\alpha, w_b \to (w-\alpha)_B} \\ &\quad = \langle \alpha, w \rangle D_w + G^{w+\alpha}_{\Gamma\Delta} (G^w)^{-1}_{cd} N_{-\alpha, (w+\alpha)_\Gamma \to w_c} N^*_{-\alpha, (w-\alpha)_\Delta \to w_d}. \end{aligned} \tag{24}$$

This expression is easily iterated along a sequence of lowerings with the same ladder operator:

$$
\begin{aligned}
&(G^{w})_{ab}\ (G^{w-\alpha})^{-1}_{AB}\ N_{-\alpha,\,w_a\to(w-\alpha)_A}\,N^{*}_{-\alpha,\,w_b\to(w-\alpha)_B}\\
&\quad=\ \langle\,\alpha,w\,\rangle\,D_w+\langle\,\alpha,w+\alpha\,\rangle\,D_{w+\alpha}+\ G^{w+2\alpha}_{\gamma\delta}\,(G^{w+\alpha})^{-1}_{\Gamma\Delta}\,N_{-\alpha,\,(w+2\alpha)_\gamma\to(w+\alpha)_\Gamma}\,N^{*}_{-\alpha,\,(w+2\alpha)_\delta\to(w+\alpha)_\Delta}\\
&\quad=\ \langle\,\alpha,w\,\rangle\,D_w+\langle\,\alpha,w+\alpha\,\rangle\,D_{w+\alpha}+\ldots+\langle\,\alpha,w+k\alpha\,\rangle\,D_{w+k\alpha}\,, \qquad (25)
\end{aligned}
$$

where $(w+k\alpha)$ is the highest weight in the $SU(2)$ subgroup chain (w), $(w+\alpha)$, $(w+2\alpha)$, $\ldots$ present in the weight system.

Finally, for completeness, when raising operators are applied, eq.(23) can be written as

$$
\begin{aligned}
&(G^{w+\alpha})^{-1}_{\Gamma\Delta}\quad N_{\alpha,\,w_a\to(w+\alpha)_\Gamma}\,N^{*}_{\alpha,\,w_a\to(w+\alpha)_\Delta}\\
&\qquad=\ -\langle\,\alpha,w\,\rangle+\ G^{w-\alpha}_{CD}\,(G^{w})^{-1}_{ac}(G^{w})^{-1}_{ad}\,N_{\alpha,\,(w-\alpha)_C\to w_c}\,N^{*}_{\alpha,\,(w-\alpha)_D\to w_d}. \qquad (26)
\end{aligned}
$$

Special Cases: Lowering within basis states

Consider a series of states connected by repeated application of the same lowering operator $E_{-\alpha}$. Choose any states with degenerate weights obtained in this series as part of the basis for the degenerate weights and label these states by i. For the sequence: $\ldots,(w+\alpha)_i, w_i, (w-\alpha)_i,\ldots$ the generalized recursion relation (23) reduces to:

$$
|\ N_{-\alpha,\,w_i\to(w-\alpha)_i}\ |^2=\ \langle\,\alpha,w\,\rangle+\ G^{w+\alpha}_{\Gamma\Delta}(G^{w})^{-1}_{ic}(G^{w})^{-1}_{id}N_{-\alpha,\,(w+\alpha)_\Gamma\to w_c}N^{*}_{-\alpha,\,(w+\alpha)_\Delta\to w_d}\,. \qquad (27)
$$

When $(w+\alpha)_i$ is the only state which can be lowered by $E_{-\alpha}$ to obtain a state of weight w we get

$$
|\ N_{-\alpha,\,w_i\to(w-\alpha)_i}\ |^2=\langle\,\alpha,w\,\rangle+\ G^{(w+\alpha)}_{ii}|N_{-\alpha,\,(w+\alpha)_i\to w_i}|^2. \qquad (28)
$$

This includes the case of a non-degenerate $(w+\alpha)$ weight subspace. When $(w+\alpha)$ is non-degenerate $G^{(w+\alpha)}_{ii}=1$, which further simplifies the above relation.

A special case of interest is lowering the degenerate zero weight states of the adjoint representation which correspond to the Cartan sub-algebra. These degenerate weight states can be labeled $|\,(0)_i\,\rangle$ where the i-th degenerate weight is obtained by $E_{-\alpha_i}\,|\,\alpha_i\,\rangle\propto|\,(0)_i\,\rangle$. This basis, however, is not orthogonal. When lowering such a basis state the general formula (23) reduces to:

$$
|\ N_{-\alpha_i,(0)_j\to(-\alpha_i)}\ |^2=\ \left[(G^{(0)})^{-1}_{ji}\right]^2\ |\ N_{-\alpha_i,\,(\alpha_i)\to(0)_i}\ |^2=\ \langle(0)_j\,|\,(0)_i\rangle^2\quad|\ N_{-\alpha_i,\,(\alpha_i)\to(0)_i}\ |^2\,. \qquad (29)
$$

This result is consistent with formula (3) in section 2 when applied to the zero weight states of the **78** in E_6.

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[13] It is not always possible to choose a single basis for all of the states of degenerate weights in a representation such that lowering a particular state results in a particluar state of lower weight (as opposed to a linear combination of weights) for all possible lowerings. The simplest example of this is the 27 dimensional representation of $SU(3)$.

Table 1: **Bases in the dominant weight subspaces of the 1728-dimensional** (100001) **irrep.**

Weight state	Lowering path	Weight state	Lowering path
$\|000100_6\rangle$	6321	$\|100000_6\rangle$	65324436321
$\|000100_3\rangle$	3621	$\|100000_7\rangle$	64534236321
$\|000100_2\rangle$	2361	$\|100000_8\rangle$	63214534236
$\|000100_1\rangle$	1236	$\|100000_9\rangle$	63243654321
		$\|100000_{10}\rangle$	53624436321
		$\|100000_{11}\rangle$	54321634236
$\|100000_1\rangle$	12364534236	$\|100000_{12}\rangle$	32164534236
$\|100000_2\rangle$	21364534236	$\|100000_{13}\rangle$	32643654321
$\|100000_3\rangle$	25364436321	$\|100000_{14}\rangle$	32643254361
$\|100000_4\rangle$	24534636321	$\|100000_{15}\rangle$	45346236321
$\|100000_5\rangle$	23643254361	$\|100000_{16}\rangle$	45321634236

Table 2: **Bases in the dominant weight subspaces of the** (100100) **irrep, the $\overline{\mathbf{7371}}$.** $\vert 000010_n\rangle$ states are marked $\vert\overline{F}_n\rangle$ for brevity.

Weight state	Lowering path	Lowering paths to (000010) weight states			
$\vert 000011_4\rangle$	4321	$\vert\overline{F}_1\rangle$	514362236434321	$\vert\overline{F}_{23}\rangle$	645342136234321
$\vert 000011_3\rangle$	3421	$\vert\overline{F}_2\rangle$	563214436234321	$\vert\overline{F}_{24}\rangle$	643452136234321
$\vert 000011_2\rangle$	2341	$\vert\overline{F}_3\rangle$	536214436234321	$\vert\overline{F}_{25}\rangle$	643621345234321
$\vert 000011_1\rangle$	1234	$\vert\overline{F}_4\rangle$	523614436234321	$\vert\overline{F}_{26}\rangle$	636231245434321
$\vert 200000_6\rangle$	6345234	$\vert\overline{F}_5\rangle$	145362236434321	$\vert\overline{F}_{27}\rangle$	633221143645234
$\vert 200000_3\rangle$	3645234	$\vert\overline{F}_6\rangle$	146234536234321	$\vert\overline{F}_{28}\rangle$	632145364234321
$\vert 200000_4\rangle$	4365234	$\vert\overline{F}_7\rangle$	143621236345234	$\vert\overline{F}_{29}\rangle$	344523126634321
$\vert 200000_5\rangle$	5436234	$\vert\overline{F}_8\rangle$	143622336435421	$\vert\overline{F}_{30}\rangle$	343221166345234
$\vert 200000_2\rangle$	2364534	$\vert\overline{F}_9\rangle$	162363245434321	$\vert\overline{F}_{31}\rangle$	345234126634321
$\vert 010000_1\rangle$	23645341	$\vert\overline{F}_{10}\rangle$	162332143645234	$\vert\overline{F}_{32}\rangle$	345216321436234
$\vert 010000_2\rangle$	32645341	$\vert\overline{F}_{11}\rangle$	162332435644321	$\vert\overline{F}_{33}\rangle$	342231166345234
$\vert 010000_3\rangle$	31645234	$\vert\overline{F}_{12}\rangle$	134562236434321	$\vert\overline{F}_{34}\rangle$	342345126634321
$\vert 010000_4\rangle$	35644321	$\vert\overline{F}_{13}\rangle$	134632364523421	$\vert\overline{F}_{35}\rangle$	342312632645341
$\vert 010000_5\rangle$	36435421	$\vert\overline{F}_{14}\rangle$	134621236345234	$\vert\overline{F}_{36}\rangle$	342163423546321
$\vert 010000_6\rangle$	61345234	$\vert\overline{F}_{15}\rangle$	134622336435421	$\vert\overline{F}_{37}\rangle$	245134326634321
$\vert 010000_7\rangle$	65434321	$\vert\overline{F}_{16}\rangle$	121364236345234	$\vert\overline{F}_{38}\rangle$	241345326634321
$\vert 010000_8\rangle$	64534321	$\vert\overline{F}_{17}\rangle$	122334546634321	$\vert\overline{F}_{39}\rangle$	241332166345234
$\vert 010000_9\rangle$	63245341	$\vert\overline{F}_{18}\rangle$	123456321436234	$\vert\overline{F}_{40}\rangle$	213245346634321
$\vert 010000_{10}\rangle$	41365234	$\vert\overline{F}_{19}\rangle$	624134536234321	$\vert\overline{F}_{41}\rangle$	213243632645341
$\vert 010000_{11}\rangle$	43265341	$\vert\overline{F}_{20}\rangle$	621363245434321	$\vert\overline{F}_{42}\rangle$	213456321436234
$\vert 010000_{12}\rangle$	43546321	$\vert\overline{F}_{21}\rangle$	621332143645234	$\vert\overline{F}_{43}\rangle$	453423126634321
$\vert 010000_{13}\rangle$	51436234	$\vert\overline{F}_{22}\rangle$	621332435644321	$\vert\overline{F}_{44}\rangle$	432163423546321
$\vert 010000_{14}\rangle$	54326341				
$\vert 010000_{15}\rangle$	12364534				

Table 3: **Bases in the dominant weight subspaces of the** (100020) **irrep, the $\overline{\mathbf{7722}}$.** (100100) weight is left out as trivial. $|000010_n\rangle$ states are marked $|\overline{F}_n\rangle$ for brevity.

Weight state	Lowering path	Lowering paths to (000010) weight states			
$\lvert 000011_5\rangle$	54321	$\lvert\overline{F}_1\rangle$	5632144362345321	$\lvert\overline{F}_{21}\rangle$	3164213623452345
$\lvert 000011_4\rangle$	45321	$\lvert\overline{F}_2\rangle$	5362144362345321	$\lvert\overline{F}_{22}\rangle$	3164223645345321
$\lvert 000011_3\rangle$	34521	$\lvert\overline{F}_3\rangle$	5236144362345321	$\lvert\overline{F}_{23}\rangle$	3445231266345321
$\lvert 000011_2\rangle$	23451	$\lvert\overline{F}_4\rangle$	5123644362345321	$\lvert\overline{F}_{24}\rangle$	3452341266345321
$\lvert 000011_1\rangle$	12345	$\lvert\overline{F}_5\rangle$	5432163452364321	$\lvert\overline{F}_{25}\rangle$	3452163452364321
		$\lvert\overline{F}_6\rangle$	6421345362345321	$\lvert\overline{F}_{26}\rangle$	3452163421362345
		$\lvert\overline{F}_7\rangle$	6412345362345321	$\lvert\overline{F}_{27}\rangle$	3422636345123451
$\lvert 200000_6\rangle$	63452345	$\lvert\overline{F}_8\rangle$	6453421362345321	$\lvert\overline{F}_{28}\rangle$	3423451266345321
$\lvert 200000_3\rangle$	36452345	$\lvert\overline{F}_9\rangle$	6434521362345321	$\lvert\overline{F}_{29}\rangle$	3423126633454521
$\lvert 200000_4\rangle$	43652345	$\lvert\overline{F}_{10}\rangle$	6212363343454521	$\lvert\overline{F}_{30}\rangle$	2451343266345321
$\lvert 200000_5\rangle$	54362345	$\lvert\overline{F}_{11}\rangle$	6213632343454521	$\lvert\overline{F}_{31}\rangle$	2411363623452345
		$\lvert\overline{F}_{12}\rangle$	6213321436452345	$\lvert\overline{F}_{32}\rangle$	2413453266345321
		$\lvert\overline{F}_{13}\rangle$	6213324356454321	$\lvert\overline{F}_{33}\rangle$	2413322663453451
$\lvert 010000_1\rangle$	263453451	$\lvert\overline{F}_{14}\rangle$	6136232343454521	$\lvert\overline{F}_{34}\rangle$	2132453466345321
$\lvert 010000_2\rangle$	163452345	$\lvert\overline{F}_{15}\rangle$	6134522363454321	$\lvert\overline{F}_{35}\rangle$	2136453421362345
$\lvert 010000_3\rangle$	136452345	$\lvert\overline{F}_{16}\rangle$	6134213623452345	$\lvert\overline{F}_{36}\rangle$	1453622363454321
$\lvert 010000_4\rangle$	143652345	$\lvert\overline{F}_{17}\rangle$	6334542361234521	$\lvert\overline{F}_{37}\rangle$	1423453266345321
$\lvert 010000_5\rangle$	154362345	$\lvert\overline{F}_{18}\rangle$	6321453623454321	$\lvert\overline{F}_{38}\rangle$	1236453421362345
$\lvert 010000_6\rangle$	653454321	$\lvert\overline{F}_{19}\rangle$	3164362123452345	$\lvert\overline{F}_{39}\rangle$	4534231266345321
$\lvert 010000_7\rangle$	645345321	$\lvert\overline{F}_{20}\rangle$	3164522363454321	$\lvert\overline{F}_{40}\rangle$	4532163452364321
$\lvert 010000_8\rangle$	633454521				
$\lvert 010000_9\rangle$	326453451				
$\lvert 010000_{10}\rangle$	356454321				
$\lvert 010000_{11}\rangle$	344655321				
$\lvert 010000_{12}\rangle$	455364321				
$\lvert 010000_{13}\rangle$	453263451				
$\lvert 010000_{14}\rangle$	543263451				

Table 4: **Bases in the dominant weight subspaces of the** (110000) **irrep, the** $\overline{\mathbf{5824}}$**.**

Weight state	Lowering path
$\lvert 001000_2\rangle$	21
$\lvert 001000_1\rangle$	12
$\lvert 100010_1\rangle$	1236432
$\lvert 100010_2\rangle$	2136432
$\lvert 100010_3\rangle$	2364321
$\lvert 100010_4\rangle$	6433221
$\lvert 100010_5\rangle$	6321432
$\lvert 100010_6\rangle$	4321632
$\lvert 100010_7\rangle$	3216432
$\lvert 100010_8\rangle$	3264321
$\lvert 000001_1\rangle$	653214433221
$\lvert 000001_2\rangle$	645342133221
$\lvert 000001_3\rangle$	536214433221
$\lvert 000001_4\rangle$	532144321632
$\lvert 000001_5\rangle$	523614433221
$\lvert 000001_6\rangle$	523144321632
$\lvert 000001_7\rangle$	512364433221
$\lvert 000001_8\rangle$	512344321632
$\lvert 000001_9\rangle$	544362133221
$\lvert 000001_{10}\rangle$	415321236432
$\lvert 000001_{11}\rangle$	415322364321
$\lvert 000001_{12}\rangle$	425132136432
$\lvert 000001_{13}\rangle$	425132364321
$\lvert 000001_{14}\rangle$	433654232121
$\lvert 000001_{15}\rangle$	436542133221
$\lvert 000001_{16}\rangle$	364354232121
$\lvert 000001_{17}\rangle$	314521236432
$\lvert 000001_{18}\rangle$	314522364321
$\lvert 000001_{19}\rangle$	322436543121
$\lvert 000001_{20}\rangle$	321432654321
$\lvert 000001_{21}\rangle$	211342365432
$\lvert 000001_{22}\rangle$	213243654321
$\lvert 000001_{23}\rangle$	213645321432
$\lvert 000001_{24}\rangle$	123645321432

Lowering paths to zero weight states			
$\lvert 0_1\rangle$	65241364345236342133221	$\lvert 0_{33}\rangle$	51453643212233664433221
$\lvert 0_2\rangle$	65241363344521322364321	$\lvert 0_{34}\rangle$	51436213452233664433221
$\lvert 0_3\rangle$	65142364345236342133221	$\lvert 0_{35}\rangle$	51436213422334566321432
$\lvert 0_4\rangle$	65142363344521322364321	$\lvert 0_{36}\rangle$	53623124536436214433221
$\lvert 0_5\rangle$	65362312453436214433221	$\lvert 0_{37}\rangle$	53621453623436214433221
$\lvert 0_6\rangle$	65321453623436214433221	$\lvert 0_{38}\rangle$	53621443632314521236432
$\lvert 0_7\rangle$	65321443632314521236432	$\lvert 0_{39}\rangle$	53621443632314522364321
$\lvert 0_8\rangle$	65321443632314522364321	$\lvert 0_{40}\rangle$	54433221166345322364321
$\lvert 0_9\rangle$	62451364345236342133221	$\lvert 0_{41}\rangle$	24512345632133664433221
$\lvert 0_{10}\rangle$	62451363344521322364321	$\lvert 0_{42}\rangle$	24513245632133664433221
$\lvert 0_{11}\rangle$	62113344223366554433221	$\lvert 0_{43}\rangle$	24513246321334566321432
$\lvert 0_{12}\rangle$	62136324436321554433221	$\lvert 0_{44}\rangle$	24513213466345321236432
$\lvert 0_{13}\rangle$	62133221443366554433221	$\lvert 0_{45}\rangle$	24513213466345322364321
$\lvert 0_{14}\rangle$	62133214432635544321632	$\lvert 0_{46}\rangle$	23312435454634216633221
$\lvert 0_{15}\rangle$	64152364345236342133221	$\lvert 0_{47}\rangle$	23245341166345321236432
$\lvert 0_{16}\rangle$	64152363344521322364321	$\lvert 0_{48}\rangle$	23245341166345322364321
$\lvert 0_{17}\rangle$	64536421345236342133221	$\lvert 0_{49}\rangle$	23611435422334566321432
$\lvert 0_{18}\rangle$	64536213344521322364321	$\lvert 0_{50}\rangle$	23612344321635544321632
$\lvert 0_{19}\rangle$	64534532364312364232121	$\lvert 0_{51}\rangle$	41534563212233664433221
$\lvert 0_{20}\rangle$	61236324436321554433221	$\lvert 0_{52}\rangle$	41534632122334566321432
$\lvert 0_{21}\rangle$	61233221443366554433221	$\lvert 0_{53}\rangle$	41536213452233664433221
$\lvert 0_{22}\rangle$	61233214432635544321632	$\lvert 0_{54}\rangle$	41536213422334566321432
$\lvert 0_{23}\rangle$	63623124436321554433221	$\lvert 0_{55}\rangle$	45334221166345321236432
$\lvert 0_{24}\rangle$	63322114432635544321632	$\lvert 0_{56}\rangle$	45334221166345322364321
$\lvert 0_{25}\rangle$	52331266453436214433221	$\lvert 0_{57}\rangle$	45345632364312364232121
$\lvert 0_{26}\rangle$	52361236453436214433221	$\lvert 0_{58}\rangle$	45346323465121322364321
$\lvert 0_{27}\rangle$	52361234536436214433221	$\lvert 0_{59}\rangle$	13456342122334566321432
$\lvert 0_{28}\rangle$	52361453623436214433221	$\lvert 0_{60}\rangle$	13456213452233664433221
$\lvert 0_{29}\rangle$	52361443632314521236432	$\lvert 0_{61}\rangle$	13456213422334566321432
$\lvert 0_{30}\rangle$	52361443632314522364321	$\lvert 0_{62}\rangle$	13456223645363214433221
$\lvert 0_{31}\rangle$	51362362453436214433221	$\lvert 0_{63}\rangle$	36231244332166554433221
$\lvert 0_{32}\rangle$	51362324536436214433221	$\lvert 0_{64}\rangle$	36231244321635544321632

Table 5: **Bases in the dominant weight subspaces of the 3003-dimensional** (300000) **irrep.** (110000) weight is left out as trivial.

Weight state	Lowering path	Weight state	Lowering path
$\|001000\rangle$	211	$\|000000_1\rangle$	65241364345332364322211 1
		$\|000000_2\rangle$	651423643453323643222111
		$\|000000_3\rangle$	653623124536444333222111
$\|100010_6\rangle$	64332211	$\|000000_4\rangle$	653214436323145236432211
$\|100010_3\rangle$	36432211	$\|000000_5\rangle$	624513643453323643222111
$\|100010_2\rangle$	23643211	$\|000000_6\rangle$	621134342362365544332211
$\|100010_1\rangle$	12364321	$\|000000_7\rangle$	621332243643655443322111
		$\|000000_8\rangle$	641523643453323643222111
		$\|000000_9\rangle$	645364332345221236432111
$\|000001_1\rangle$	6544333222111	$\|000000_{10}\rangle$	612332243643655443322111
$\|000001_2\rangle$	5364433222111	$\|000000_{11}\rangle$	524536361236444333222111
$\|000001_3\rangle$	5236443322111	$\|000000_{12}\rangle$	524133214663452364332211
$\|000001_4\rangle$	5123644332211	$\|000000_{13}\rangle$	524133245434666333222111
$\|000001_5\rangle$	4251364332211	$\|000000_{14}\rangle$	513645236236444333222111
$\|000001_6\rangle$	4152364332211	$\|000000_{15}\rangle$	514362134523623644332211
$\|000001_7\rangle$	4536433222111	$\|000000_{16}\rangle$	536214436323145236432211
$\|000001_8\rangle$	3145236432211	$\|000000_{17}\rangle$	245133214663452364332211
$\|000001_9\rangle$	3452364322111	$\|000000_{18}\rangle$	245133245434666333222111
$\|000001_{10}\rangle$	2345123643211	$\|000000_{19}\rangle$	213216324364365544332211
		$\|000000_{20}\rangle$	415345234234666333222111
		$\|000000_{21}\rangle$	415362134523623644332211
		$\|000000_{22}\rangle$	453342211663452364332211
		$\|000000_{23}\rangle$	134562134523623644332211
		$\|000000_{24}\rangle$	345634234653221236432111

Table 6: **CG coefficients for (000100) dominant weight in (100000)⊗(000001).** Each entry should be divided by the respective number in the last row to keep the states normalized to 1.

	(100001)				*(000100)*
	$\lvert 000100_6\rangle$	$\lvert 000100_3\rangle$	$\lvert 000100_2\rangle$	$\lvert 000100_1\rangle$	$\lvert 000100\rangle$
$\lvert 00010\bar{1}\rangle\,\lvert 000001\rangle$	1				1
$\lvert 00\bar{1}101\rangle\,\lvert 00100\bar{1}\rangle$	1	1			-1
$\lvert 0\bar{1}1000\rangle\,\lvert 01\bar{1}100\rangle$		1	1		1
$\lvert \bar{1}10000\rangle\,\lvert 1\bar{1}0100\rangle$			1	1	-1
$\lvert 100000\rangle\,\lvert \bar{1}00100\rangle$				1	1
	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{5}$

Table 7: **CG coefficients for (100000) dominant weight in (100000)⊗(000001).** The fundamental (100000) irrep is marked as F, and its highest weight state as $\lvert \mathrm{F}\rangle$˙$\lvert \mathrm{n}\rangle$ is an abbreviation for $\lvert 100000_n\rangle$. Numbering of the degenerate states is consistent with table 1 and eqs.(8–9). Each CGC should be divided by the respective number in the last row to maintain $\langle n\,\vert\, n\rangle = 1$.

	(100001)																*(000100)*					*F*
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert 7\rangle$	$\lvert 8\rangle$	$\lvert 9\rangle$	$\lvert 10\rangle$	$\lvert 11\rangle$	$\lvert 12\rangle$	$\lvert 13\rangle$	$\lvert 14\rangle$	$\lvert 15\rangle$	$\lvert 16\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert \mathrm{F}\rangle$
$\lvert 100000\rangle\,\lvert 0_1\rangle$	$\sqrt{2}$																					$\sqrt{32}$
$\lvert 100000\rangle\,\lvert 0_2\rangle$		$\sqrt{2}$															$\sqrt{2}$					-$\sqrt{50}$
$\lvert 100000\rangle\,\lvert 0_3\rangle$												$\sqrt{2}$						$\sqrt{2}$				$\sqrt{72}$
$\lvert 100000\rangle\,\lvert 0_4\rangle$																$\sqrt{2}$			$\sqrt{2}$			-$\sqrt{32}$
$\lvert 100000\rangle\,\lvert 0_5\rangle$											$\sqrt{2}$									$\sqrt{2}$		$\sqrt{8}$
$\lvert 100000\rangle\,\lvert 0_6\rangle$								$\sqrt{2}$													$\sqrt{2}$	-$\sqrt{18}$
$\lvert \bar{1}10000\rangle\,\lvert 2\bar{1}0000\rangle$	1	1			1												-1					-3
$\lvert 0\bar{1}1000\rangle\,\lvert 11\bar{1}000\rangle$		1			1							1		1			-1	-1				3
$\lvert 00\bar{1}101\rangle\,\lvert 101\bar{1}0\bar{1}\rangle$							1	1				1		1		1		-1	-1		-1	-3
$\lvert 000\bar{1}11\rangle\,\lvert 1001\bar{1}\bar{1}\rangle$						1	1				1					1			-1	-1	1	3
$\lvert 00010\bar{1}\rangle\,\lvert 100\bar{1}01\rangle$							1	1							1				1		-1	3
$\lvert 0000\bar{1}1\rangle\,\lvert 10001\bar{1}\rangle$						1			1		1									-1	-1	-3
$\lvert 001\bar{1}1\bar{1}\rangle\,\lvert 10\bar{1}1\bar{1}1\rangle$						1	1			1				1	1			1	1	1	1	-3
$\lvert 0010\bar{1}\bar{1}\rangle\,\lvert 10\bar{1}011\rangle$						1			1	1			1					-1		1	-1	3
$\lvert 01\bar{1}010\rangle\,\lvert 1\bar{1}10\bar{1}0\rangle$			1		1					1				1			1	1		-1		3
$\lvert 01\bar{1}1\bar{1}0\rangle\,\lvert 1\bar{1}1\bar{1}10\rangle$			1	1						1			1		1		-1	-1	-1	-1		-3
$\lvert 1\bar{1}0010\rangle\,\lvert 0100\bar{1}0\rangle$			1		1												1			1		-3
$\lvert 010\bar{1}00\rangle\,\lvert 1\bar{1}0100\rangle$				1											1		1		-1			3
$\lvert 1\bar{1}01\bar{1}0\rangle\,\lvert 010\bar{1}10\rangle$			1	1													-1		1	1		3
$\lvert 1\bar{1}1\bar{1}00\rangle\,\lvert 01\bar{1}100\rangle$				1									1				1	1	1			-3
$\lvert 10\bar{1}001\rangle\,\lvert 00100\bar{1}\rangle$									1				1					1			1	3
$\lvert 10000\bar{1}\rangle\,\lvert 000001\rangle$									1												1	-3
	$\sqrt{3}$	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{156}$

Table 8: **CG coefficients for (000011) dominant weight in (100000)⊗(000100).** Each entry should be divided by the respective number in the last row to keep the states normalized to 1.

	(100100)				*(000011)*
	$\lvert 000011_4\rangle$	$\lvert 000011_3\rangle$	$\lvert 000011_2\rangle$	$\lvert 000011_1\rangle$	$\lvert 000011\rangle$
$\lvert 000\bar{1}11\rangle\ \lvert 000100\rangle$	1				1
$\lvert 00\bar{1}101\rangle\ \lvert 001\bar{1}10\rangle$	1	1			-1
$\lvert 0\bar{1}1000\rangle\ \lvert 01\bar{1}011\rangle$		1	1		1
$\lvert \bar{1}10000\rangle\ \lvert 1\bar{1}0011\rangle$			1	1	-1
$\lvert 100000\rangle\ \lvert \bar{1}00011\rangle$				1	1
	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{5}$

Table 9: **CG coefficients for (010000) dominant weight in (100000)⊗(000100).** The (010000) irrep is marked as $\overline{G}$, and its highest weight state as $|\overline{G}\rangle$. |n⟩ is an abbreviation for $|010000_n\rangle$. Numbering of the degenerate states is consistent with tables 2 and 1. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(100100)															*(000011)*				$\overline{G}$
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|6\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|\overline{G}\rangle$
$\|100000\rangle\|\bar{1}10000_2\rangle$															1					5
$\|100000\rangle\|\bar{1}10000_3\rangle$			1														$\sqrt{2}$			-6
$\|100000\rangle\|\bar{1}10000_4\rangle$										1								$\sqrt{2}$		4
$\|100000\rangle\|\bar{1}10000_5\rangle$													1						$\sqrt{2}$	-2
$\|100000\rangle\|\bar{1}10000_6\rangle$						1										$\sqrt{2}$				3
$\|\bar{1}10000\rangle\|100000_2\rangle$	$\sqrt{2}$														1					-5
$\|\bar{1}10000\rangle\|100000_3\rangle$		$\sqrt{2}$	1														$-\sqrt{2}$			6
$\|\bar{1}10000\rangle\|100000_4\rangle$										1	$\sqrt{2}$							$-\sqrt{2}$		-4
$\|\bar{1}10000\rangle\|100000_5\rangle$													1	$\sqrt{2}$					$-\sqrt{2}$	2
$\|\bar{1}10000\rangle\|100000_6\rangle$						1			$\sqrt{2}$							$-\sqrt{2}$				-3
$\|0\bar{1}1000\rangle\|02\bar{1}000\rangle$	1	1			1												1			$\sqrt{8}$
$\|00\bar{1}101\rangle\|011\bar{1}0\bar{1}\rangle$		1			1			1	1		1					1	1	1		$-\sqrt{8}$
$\|000\bar{1}11\rangle\|0101\bar{1}\bar{1}\rangle$							1	1			1			1		-1		1	1	$\sqrt{8}$
$\|00010\bar{1}\rangle\|010\bar{1}01\rangle$								1	1			1				1		-1		$\sqrt{8}$
$\|0000\bar{1}1\rangle\|01001\bar{1}\rangle$							1							1		1			1	$-\sqrt{8}$
$\|001\bar{1}1\bar{1}\rangle\|01\bar{1}1\bar{1}1\rangle$				1	1		1	1				1				-1	-1	-1	-1	$-\sqrt{8}$
$\|0010\bar{1}\bar{1}\rangle\|01\bar{1}011\rangle$				1			1									1	1		-1	$\sqrt{8}$
$\|01\bar{1}010\rangle\|0010\bar{1}0\rangle$				1	1												-1		1	$\sqrt{8}$
$\|01\bar{1}1\bar{1}0\rangle\|001\bar{1}10\rangle$				1								1					1	1	1	$-\sqrt{8}$
$\|010\bar{1}00\rangle\|000100\rangle$												1						1		$\sqrt{8}$
	$\sqrt{3}$	2	$\sqrt{2}$	2	2	$\sqrt{2}$	2	2	2	$\sqrt{2}$	2	2	$\sqrt{2}$	2	$\sqrt{2}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{10}$	$\sqrt{10}$	10

Table 10: **CG coefficients for (000010) dominant weight in (100000)⊗(000100).** $|n\rangle$ is an abbreviation for $|000010_n\rangle$. Numbering of the degenerate states is consistent with tables 2, and 1. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(100100)																																
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$	$\vert 21\rangle$	$\vert 22\rangle$	$\vert 23\rangle$	$\vert 24\rangle$	$\vert 25\rangle$	$\vert 26\rangle$	$\vert 27\rangle$	$\vert 28\rangle$	$\vert 29\rangle$	$\vert 30\rangle$	$\vert 31\rangle$	$\vert 32\rangle$	$\vert 33\rangle$
$\vert 100000\rangle\,\vert \bar{1}00010_2\rangle$																1																	
$\vert 100000\rangle\,\vert \bar{1}00010_3\rangle$														1																			
$\vert 100000\rangle\,\vert \bar{1}00010_4\rangle$										1																							
$\vert 100000\rangle\,\vert \bar{1}00010_5\rangle$																		1															
$\vert 100000\rangle\,\vert \bar{1}00010_6\rangle$							1																										
$\vert \bar{1}00010\rangle\,\vert 100000_2\rangle$																	1																
$\vert \bar{1}00010\rangle\,\vert 100000_3\rangle$												$\sqrt{2}$			1																		
$\vert \bar{1}00010\rangle\,\vert 100000_4\rangle$					$\sqrt{2}$			1																									
$\vert \bar{1}00010\rangle\,\vert 100000_5\rangle$	$\sqrt{2}$																	1															
$\vert \bar{1}00010\rangle\,\vert 100000_6\rangle$									1		$\sqrt{2}$																						
$\vert \bar{1}10000\rangle\,\vert 1\bar{1}0010_2\rangle$																1																	
$\vert \bar{1}10000\rangle\,\vert 1\bar{1}0010_3\rangle$														1																			1
$\vert \bar{1}10000\rangle\,\vert 1\bar{1}0010_4\rangle$										1											1												
$\vert \bar{1}10000\rangle\,\vert 1\bar{1}0010_5\rangle$																		1															
$\vert \bar{1}10000\rangle\,\vert 1\bar{1}0010_6\rangle$							1																										
$\vert 1\bar{1}0010\rangle\,\vert \bar{1}10000_2\rangle$																	1																
$\vert 1\bar{1}0010\rangle\,\vert \bar{1}10000_3\rangle$												$\sqrt{2}$			1																$\sqrt{2}$		
$\vert 1\bar{1}0010\rangle\,\vert \bar{1}10000_4\rangle$					$\sqrt{2}$			1																									
$\vert 1\bar{1}0010\rangle\,\vert \bar{1}10000_5\rangle$	$\sqrt{2}$			$\sqrt{2}$														1															
$\vert 1\bar{1}0010\rangle\,\vert \bar{1}10000_6\rangle$									1		$\sqrt{2}$									1		$\sqrt{2}$						$\sqrt{2}$					
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}010_2\rangle$													1	1		1																	
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}010_3\rangle$																																	1
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}010_4\rangle$																					1						1						
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}010_5\rangle$																																1	
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}010_6\rangle$																														1			
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}1000_2\rangle$													1		1		1																
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}1000_3\rangle$																																	
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}1000_4\rangle$																													1		$\sqrt{2}$		
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}1000_5\rangle$			$\sqrt{2}$	$\sqrt{2}$																												1	
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}1000_6\rangle$																				1		$\sqrt{2}$				1							
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}1\bar{1}_2\rangle$						1	1			1			1	1																			
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}1\bar{1}_3\rangle$																			1		1												1
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}1\bar{1}_4\rangle$																								1			1						
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}1\bar{1}_5\rangle$																							1									1	
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}1\bar{1}_6\rangle$																									1					1			
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}101_2\rangle$						1		1	1				1		1																		
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}101_3\rangle$																			1	1													
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}101_4\rangle$																								1				$\sqrt{2}$	1		$\sqrt{2}$		
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}101_5\rangle$		$\sqrt{2}$	$\sqrt{2}$																				1									1	
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}101_6\rangle$																									1	1							
$\vert 000\bar{1}11\rangle\,\vert 00010\bar{1}_2\rangle$						1	1		1																								
$\vert 000\bar{1}11\rangle\,\vert 00010\bar{1}_3\rangle$																			1	1													
$\vert 000\bar{1}11\rangle\,\vert 00010\bar{1}_4\rangle$																								1				$\sqrt{2}$					
$\vert 000\bar{1}11\rangle\,\vert 00010\bar{1}_5\rangle$		$\sqrt{2}$																					1										
$\vert 000\bar{1}11\rangle\,\vert 00010\bar{1}_6\rangle$																									1	1				1			
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}11_2\rangle$						1		1		1								1															
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}11_3\rangle$																			1		1												
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}11_4\rangle$																								1			1		1			1	
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}11_5\rangle$																							1										
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}11_6\rangle$																									1								
$\vert \bar{1}001\bar{1}0\rangle\,\vert 100\bar{1}20\rangle$	1				1																												
$\vert 1\bar{1}01\bar{1}0\rangle\,\vert \bar{1}10\bar{1}20\rangle$	1			1	1																												
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}20\rangle$			1	1																											1		
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}021\rangle$		1	1																									1			1		
$\vert 0000\bar{1}1\rangle\,\vert 00002\bar{1}\rangle$		1																										1					
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0110\rangle$																																	
$\vert 1\bar{1}1\bar{1}00\rangle\,\vert \bar{1}1\bar{1}110\rangle$					1							1																			1		
$\vert \bar{1}01\bar{1}00\rangle\,\vert 10\bar{1}110\rangle$					1							1																					
$\vert 10\bar{1}001\rangle\,\vert \bar{1}0101\bar{1}\rangle$											1	1																1			1		
$\vert \bar{1}1\bar{1}001\rangle\,\vert 1\bar{1}101\bar{1}\rangle$											1	1										1											
$\vert 10000\bar{1}\rangle\,\vert \bar{1}00011\rangle$											1																	1					
$\vert 0\bar{1}0001\rangle\,\vert 01001\bar{1}\rangle$																						1											
$\vert \bar{1}1000\bar{1}\rangle\,\vert 1\bar{1}0011\rangle$											1											1											
$\vert 0\bar{1}100\bar{1}\rangle\,\vert 01\bar{1}011\rangle$																						1											
$\vert 00\bar{1}100\rangle\,\vert 001\bar{1}10\rangle$																																	
$\vert 000\bar{1}10\rangle\,\vert 000100\rangle$																																	
	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{8}$	2	2	2	2	2	$\sqrt{8}$	$\sqrt{8}$	2	2	2	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{5}$	2	2	2	$\sqrt{8}$	2	2	2	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{5}$	$\sqrt{3}$

Table 11: **CG coefficients for (000010) dominant weight in (100000)⊗(000100).** The fundamental (100000) irrep is marked as $\overline{F}$, and its highest weight state as $\vert\,\overline{F}\rangle$. $\vert$n$\rangle$ is an abbreviation for $\vert 000010_n\rangle$. Numbering of the degenerate states is consistent with tables 2, 1, and eqs.(8–9). Each CGC should be divided by the respective number in the last row to maintain $\langle n\vert n\rangle = 1$.

	(100100) continued from table 10											*(000011)*																*(010000)*					$\overline{F}$
	$\vert 34\rangle$	$\vert 35\rangle$	$\vert 36\rangle$	$\vert 37\rangle$	$\vert 38\rangle$	$\vert 39\rangle$	$\vert 40\rangle$	$\vert 41\rangle$	$\vert 42\rangle$	$\vert 43\rangle$	$\vert 44\rangle$	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 6\rangle$	$\vert 3\rangle$	$\vert 2\rangle$	$\vert 1\rangle$	$\vert 4\rangle$	$\vert\overline{F}\rangle$
$\vert 100000\rangle\ \vert\bar{1}00010_2\rangle$																										$\sqrt{2}$				4	5		1
$\vert 100000\rangle\ \vert\bar{1}00010_3\rangle$																					$\sqrt{2}$								4		-6		-2
$\vert 100000\rangle\ \vert\bar{1}00010_4\rangle$																	$\sqrt{2}$											4			4		2
$\vert 100000\rangle\ \vert\bar{1}00010_5\rangle$																						$\sqrt{2}$									-2		-2
$\vert 100000\rangle\ \vert\bar{1}00010_6\rangle$														$\sqrt{2}$																	3	4	1
$\vert\bar{1}00010\rangle\ \vert 100000_2\rangle$							$\sqrt{2}$															$\sqrt{2}$				-$\sqrt{2}$	$\sqrt{2}$			4	3		1
$\vert\bar{1}00010\rangle\ \vert 100000_3\rangle$																					-$\sqrt{2}$	-$\sqrt{2}$	$\sqrt{2}$						4		-2		-2
$\vert\bar{1}00010\rangle\ \vert 100000_4\rangle$													$\sqrt{2}$	-$\sqrt{2}$								$\sqrt{2}$										4	2
$\vert\bar{1}00010\rangle\ \vert 100000_5\rangle$												$\sqrt{2}$										-$\sqrt{2}$									2		-2
$\vert\bar{1}00010\rangle\ \vert 100000_6\rangle$																	-$\sqrt{2}$		$\sqrt{2}$									4			1		1
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0010_2\rangle$								1																		-$\sqrt{2}$				1	5		-1
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0010_3\rangle$																					$\sqrt{2}$			$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$			-4	-6	-6		2
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0010_4\rangle$																	$\sqrt{2}$	$\sqrt{2}$		$\sqrt{2}$								-4		4	4		-2
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0010_5\rangle$									1													$\sqrt{2}$					$\sqrt{2}$			-2	-2		2
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0010_6\rangle$						1								$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$														3	3	-4	-1
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}10000_2\rangle$								1														$\sqrt{2}$				$\sqrt{2}$				-1	3		-1
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}10000_3\rangle$	1		$\sqrt{2}$				$\sqrt{2}$														-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$		-4	-2	-2		2
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}10000_4\rangle$				$\sqrt{2}$	1						$\sqrt{2}$		-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$						$\sqrt{2}$					$\sqrt{2}$					-4	-2
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}10000_5\rangle$									1			-$\sqrt{2}$										-$\sqrt{2}$					-$\sqrt{2}$			2	2		2
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}10000_6\rangle$																	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$								-4		1	1		-1
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}010_2\rangle$								1													-$\sqrt{2}$					-$\sqrt{2}$			1	1	-4		1
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}010_3\rangle$		1																						-$\sqrt{2}$	-$\sqrt{2}$	$\sqrt{2}$			-2	-6			-2
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}010_4\rangle$																		$\sqrt{2}$	$\sqrt{2}$	-$\sqrt{2}$					$\sqrt{2}$			4	4	4			2
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}010_5\rangle$									1														$\sqrt{2}$				$\sqrt{2}$		-2	-2			-2
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}010_6\rangle$						1									$\sqrt{2}$	-$\sqrt{2}$								$\sqrt{2}$					3	3		4	1
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}1000_2\rangle$								1													$\sqrt{2}$	-$\sqrt{2}$				$\sqrt{2}$			-1	-1	-4		1
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}1000_3\rangle$	1	1					$\sqrt{2}$																	$\sqrt{2}$	$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$		2	-2			-2
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}1000_4\rangle$	1			$\sqrt{2}$	1					$\sqrt{2}$			$\sqrt{2}$		-$\sqrt{2}$	$\sqrt{2}$							$\sqrt{2}$	-$\sqrt{2}$			$\sqrt{2}$					4	2
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}1000_5\rangle$									1			$\sqrt{2}$											-$\sqrt{2}$				-$\sqrt{2}$		2	2			-2
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}1000_6\rangle$			$\sqrt{2}$															-$\sqrt{2}$		$\sqrt{2}$					-$\sqrt{2}$			4	1	1			1
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}1\bar{1}_2\rangle$														-$\sqrt{2}$			-$\sqrt{2}$				-$\sqrt{2}$							1	1		4	1	-1
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}1\bar{1}_3\rangle$		1				1		1							-$\sqrt{2}$	-$\sqrt{2}$		-$\sqrt{2}$		-$\sqrt{2}$				-$\sqrt{2}$	-$\sqrt{2}$	-$\sqrt{2}$		-2	-2	4		-2	2
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}1\bar{1}_4\rangle$																$\sqrt{2}$			-$\sqrt{2}$	$\sqrt{2}$					$\sqrt{2}$				4			4	-2
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}1\bar{1}_5\rangle$													$\sqrt{2}$						$\sqrt{2}$				$\sqrt{2}$					-2	-2			-2	2
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}1\bar{1}_6\rangle$																$\sqrt{2}$				$\sqrt{2}$				$\sqrt{2}$				3	3			-1	-1
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}101_2\rangle$														$\sqrt{2}$			$\sqrt{2}$				$\sqrt{2}$	$\sqrt{2}$						-1	-1		4	-1	-1
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}101_3\rangle$	1	1			1			1							$\sqrt{2}$	$\sqrt{2}$		$\sqrt{2}$		$\sqrt{2}$				$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	2	2	4		2	2
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}101_4\rangle$	1															-$\sqrt{2}$			$\sqrt{2}$	-$\sqrt{2}$			$\sqrt{2}$	-$\sqrt{2}$								-4	-2
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}101_5\rangle$										$\sqrt{2}$		-$\sqrt{2}$	-$\sqrt{2}$						-$\sqrt{2}$				-$\sqrt{2}$					2	2			2	2
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}101_6\rangle$			$\sqrt{2}$								$\sqrt{2}$					-$\sqrt{2}$				-$\sqrt{2}$					-$\sqrt{2}$			-3	1			1	-1
$\vert 000\bar{1}11\rangle\ \vert 00010\bar{1}_2\rangle$														-$\sqrt{2}$			$\sqrt{2}$											-1			-4	1	1
$\vert 000\bar{1}11\rangle\ \vert 00010\bar{1}_3\rangle$						1									-$\sqrt{2}$	-$\sqrt{2}$		$\sqrt{2}$		$\sqrt{2}$								2		-4		-2	-2
$\vert 000\bar{1}11\rangle\ \vert 00010\bar{1}_4\rangle$																$\sqrt{2}$			$\sqrt{2}$	-$\sqrt{2}$												4	2
$\vert 000\bar{1}11\rangle\ \vert 00010\bar{1}_5\rangle$												$\sqrt{2}$	$\sqrt{2}$						-$\sqrt{2}$									2				-2	-2
$\vert 000\bar{1}11\rangle\ \vert 00010\bar{1}_6\rangle$		1														$\sqrt{2}$				-$\sqrt{2}$				-$\sqrt{2}$	$\sqrt{2}$			-3	-4			-1	1
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}11_2\rangle$														$\sqrt{2}$			-$\sqrt{2}$					-$\sqrt{2}$						1			-4	-1	1
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}11_3\rangle$					1				1						$\sqrt{2}$	$\sqrt{2}$		-$\sqrt{2}$		-$\sqrt{2}$							-$\sqrt{2}$	-2		-4		2	-2
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}11_4\rangle$	1	1														-$\sqrt{2}$			-$\sqrt{2}$	$\sqrt{2}$			-$\sqrt{2}$	$\sqrt{2}$	-$\sqrt{2}$				-4			-4	2
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}11_5\rangle$										$\sqrt{2}$			-$\sqrt{2}$						$\sqrt{2}$									-2				2	-2
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}11_6\rangle$											$\sqrt{2}$					-$\sqrt{2}$				$\sqrt{2}$								3				1	1
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}20\rangle$											1	1	1	1		1						1									-$\sqrt{8}$	-$\sqrt{8}$	$\sqrt{2}$
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}20\rangle$				1								-1	-1	1	1							1					1			-$\sqrt{8}$	-$\sqrt{8}$	$\sqrt{8}$	-$\sqrt{2}$
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}20\rangle$				1						1		1	1		1								1	1			1		-$\sqrt{8}$	-$\sqrt{8}$		-$\sqrt{8}$	$\sqrt{2}$
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}021\rangle$												-1							1	1			1	1				-$\sqrt{8}$	-$\sqrt{8}$				-$\sqrt{2}$
$\vert 000\bar{1}1\rangle\ \vert 00002\bar{1}\rangle$												1							1	1								-$\sqrt{8}$					$\sqrt{2}$
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0110\rangle$				1			1			1			1		1											1	-1			$\sqrt{8}$		-$\sqrt{8}$	-$\sqrt{2}$
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}110\rangle$				1			1						-1	1	1						1	-1	-1	1		1	-1		$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{2}$
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}110\rangle$			1								1		1	1		1					1	-1	1		1				-$\sqrt{8}$		$\sqrt{8}$	-$\sqrt{8}$	-$\sqrt{2}$
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0101\bar{1}\rangle$																	1		-1	1	1	1	-1	1				$\sqrt{8}$	$\sqrt{8}$		-$\sqrt{8}$		-$\sqrt{2}$
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}101\bar{1}\rangle$			1				1										1	1	1		1	1	1		1	1	1	-$\sqrt{8}$	-$\sqrt{8}$	-$\sqrt{8}$	-$\sqrt{8}$		$\sqrt{2}$
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00011\rangle$																	1		-1	1								$\sqrt{8}$			$\sqrt{8}$		$\sqrt{2}$
$\vert 0\bar{1}0001\rangle\ \vert 01001\bar{1}\rangle$							1											1								1	1	$\sqrt{8}$		-$\sqrt{8}$			-$\sqrt{2}$
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0011\rangle$																	1	1	1									-$\sqrt{8}$		$\sqrt{8}$	$\sqrt{8}$		-$\sqrt{2}$
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}011\rangle$			1															1							1			$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$			$\sqrt{2}$
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}10\rangle$			1								1					1									1				$\sqrt{8}$			$\sqrt{8}$	-$\sqrt{2}$
$\vert 000\bar{1}10\rangle\ \vert 000100\rangle$											1					1																$\sqrt{8}$	$\sqrt{2}$
	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{10}$	$\sqrt{8}$	2	2	$\sqrt{10}$	$\sqrt{6}$	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{10}$	$\sqrt{15}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{20}$	$\sqrt{200}$ each state					$\sqrt{52}$

Table 12: **CG coefficients for (000011) dominant weight in (100000)⊗(000020).** Each entry should be divided by the respective number in the last row to keep the states normalized to 1.

	(100020)					*(000011)*
	$\|000011_5\rangle$	$\|000011_4\rangle$	$\|000011_3\rangle$	$\|000011_2\rangle$	$\|000011_1\rangle$	$\|000011\rangle$
$\|0000\bar{1}1\rangle\,\|000020\rangle$	1					$-\sqrt{2}$
$\|000\bar{1}11\rangle\,\|000100\rangle$	$\sqrt{2}$	1				1
$\|00\bar{1}101\rangle\,\|001\bar{1}10\rangle$		1	1			-1
$\|0\bar{1}1000\rangle\,\|01\bar{1}011\rangle$			1	1		1
$\|\bar{1}10000\rangle\,\|1\bar{1}0011\rangle$				1	1	-1
$\|100000\rangle\,\|\bar{1}00011\rangle$					1	1
	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{7}$

Table 13: **CG coefficients for (010000) dominant weight in (100000)⊗(000020).** $|n\rangle$ is an abbreviation for $|010000_n\rangle$. Numbering of the degenerate states is consistent with tables 3, and 1. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(100020)														*(000011)*			
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|6\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$
$\|100000\rangle\,\|\bar{1}10000_3\rangle$			1													$\sqrt{2}$		
$\|100000\rangle\,\|\bar{1}10000_4\rangle$				1													$\sqrt{2}$	
$\|100000\rangle\,\|\bar{1}10000_5\rangle$					1													$\sqrt{2}$
$\|100000\rangle\,\|\bar{1}10000_6\rangle$		1													$\sqrt{2}$			
$\|\bar{1}10000\rangle\,\|100000_3\rangle$			1						$\sqrt{2}$							$-\sqrt{2}$		
$\|\bar{1}10000\rangle\,\|100000_4\rangle$				1									$\sqrt{2}$				$-\sqrt{2}$	
$\|\bar{1}10000\rangle\,\|100000_5\rangle$					1									$\sqrt{2}$				$-\sqrt{2}$
$\|\bar{1}10000\rangle\,\|100000_6\rangle$	$\sqrt{2}$	1													$-\sqrt{2}$			
$\|0\bar{1}1000\rangle\,\|02\bar{1}000\rangle$	1							1	1						2	1		
$\|00\bar{1}101\rangle\,\|011\bar{1}0\bar{1}\rangle$							1	1	1				1		-1	1	1	
$\|000\bar{1}11\rangle\,\|0101\bar{1}\bar{1}\rangle$						1	1						1	1	1		1	1
$\|00010\bar{1}\rangle\,\|010\bar{1}01\rangle$							1	1			1				-1	-2	-1	
$\|0000\bar{1}1\rangle\,\|01001\bar{1}\rangle$						1								1	-1			1
$\|001\bar{1}1\bar{1}\rangle\,\|01\bar{1}1\bar{1}1\rangle$						1	1			1	1				1	1	-1	-1
$\|0010\bar{1}\bar{1}\rangle\,\|01\bar{1}011\rangle$						1				1					-1	-1		-1
$\|01\bar{1}010\rangle\,\|0010\bar{1}0\rangle$										1	1	1				1	2	1
$\|01\bar{1}1\bar{1}0\rangle\,\|001\bar{1}10\rangle$										1		1				-1	-1	1
$\|010\bar{1}00\rangle\,\|000100\rangle$												1					-1	-2
	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	2	2	$\sqrt{3}$	2	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{3}$	2	2	$\sqrt{14}$	$\sqrt{14}$	$\sqrt{14}$	$\sqrt{14}$

Table 14: **CG coefficients for (000010) dominant weight in (100000)⊗(000020).** $|\mathrm{n}\rangle$ is an abbreviation for $|000010_n\rangle$. Numbering of the degenerate states is consistent with tables 3, and 1. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(100020)																												
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|16\rangle$	$\|17\rangle$	$\|18\rangle$	$\|19\rangle$	$\|20\rangle$	$\|21\rangle$	$\|22\rangle$	$\|23\rangle$	$\|24\rangle$	$\|25\rangle$	$\|26\rangle$	$\|27\rangle$	$\|28\rangle$	$\|29\rangle$
$\|100000\rangle\ \|\bar{1}00010_3\rangle$																					1								
$\|100000\rangle\ \|\bar{1}00010_4\rangle$																1													
$\|100000\rangle\ \|\bar{1}00010_5\rangle$																													
$\|100000\rangle\ \|\bar{1}00010_6\rangle$																													
$\|\bar{1}00010\rangle\ \|100000_3\rangle$																				$\sqrt{2}$		1			$\sqrt{2}$				
$\|\bar{1}00010\rangle\ \|100000_4\rangle$																													
$\|\bar{1}00010\rangle\ \|100000_5\rangle$				$\sqrt{2}$	2																								
$\|\bar{1}00010\rangle\ \|100000_6\rangle$										1			$\sqrt{8}$		$\sqrt{2}$														
$\|\bar{1}10000\rangle\ \|1\bar{1}0010_3\rangle$																					1						1		
$\|\bar{1}10000\rangle\ \|1\bar{1}0010_4\rangle$												1				1													
$\|\bar{1}10000\rangle\ \|1\bar{1}0010_5\rangle$																													
$\|\bar{1}10000\rangle\ \|1\bar{1}0010_6\rangle$																													
$\|1\bar{1}0010\rangle\ \|\bar{1}10000_3\rangle$																				$\sqrt{2}$		1		$\sqrt{2}$				1	
$\|1\bar{1}0010\rangle\ \|\bar{1}10000_4\rangle$																													
$\|1\bar{1}0010\rangle\ \|\bar{1}10000_5\rangle$			$\sqrt{2}$	$\sqrt{2}$																									
$\|1\bar{1}0010\rangle\ \|\bar{1}10000_6\rangle$										1	1		$\sqrt{2}$		$\sqrt{2}$			$\sqrt{2}$											
$\|0\bar{1}1000\rangle\ \|01\bar{1}010_3\rangle$																											1		1
$\|0\bar{1}1000\rangle\ \|01\bar{1}010_4\rangle$												1					1												
$\|0\bar{1}1000\rangle\ \|01\bar{1}010_5\rangle$																										1			
$\|0\bar{1}1000\rangle\ \|01\bar{1}010_6\rangle$																			1		1								
$\|01\bar{1}010\rangle\ \|0\bar{1}1000_3\rangle$																												1	1
$\|01\bar{1}010\rangle\ \|0\bar{1}1000_4\rangle$																							1	$\sqrt{2}$				1	
$\|01\bar{1}010\rangle\ \|0\bar{1}1000_5\rangle$		$\sqrt{2}$	$\sqrt{2}$																							1			
$\|01\bar{1}010\rangle\ \|0\bar{1}1000_6\rangle$										1	1		$\sqrt{2}$	1					1			3							
$\|00\bar{1}101\rangle\ \|001\bar{1}1\bar{1}_3\rangle$						1						1															1		1
$\|00\bar{1}101\rangle\ \|001\bar{1}1\bar{1}_4\rangle$									1								1												
$\|00\bar{1}101\rangle\ \|001\bar{1}1\bar{1}_5\rangle$								1																		1			
$\|00\bar{1}101\rangle\ \|001\bar{1}1\bar{1}_6\rangle$																													
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}101_3\rangle$						1					1																	1	1
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}101_4\rangle$									1									$\sqrt{2}$					1	$\sqrt{2}$				1	
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}101_5\rangle$	$\sqrt{2}$	$\sqrt{2}$						1																		1			
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}101_6\rangle$							1							1					1			3							
$\|000\bar{1}11\rangle\ \|00010\bar{1}_3\rangle$						1					1																		
$\|000\bar{1}11\rangle\ \|00010\bar{1}_4\rangle$									1									$\sqrt{2}$											
$\|000\bar{1}11\rangle\ \|00010\bar{1}_5\rangle$	$\sqrt{2}$							1																					
$\|000\bar{1}11\rangle\ \|00010\bar{1}_6\rangle$							1							1					2			2							1
$\|00010\bar{1}\rangle\ \|000\bar{1}11_3\rangle$						1						1																	
$\|00010\bar{1}\rangle\ \|000\bar{1}11_4\rangle$									1								1						1			1		1	1
$\|00010\bar{1}\rangle\ \|000\bar{1}11_5\rangle$								1																					
$\|00010\bar{1}\rangle\ \|000\bar{1}11_6\rangle$							1									1													
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}20\rangle$				1	$\sqrt{2}$																								
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}20\rangle$			1	1																									
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}20\rangle$		1	1																					1					
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}021\rangle$	1	1																1						1					
$\|0000\bar{1}1\rangle\ \|00002\bar{1}\rangle$	1																	1											
$\|010\bar{1}00\rangle\ \|0\bar{1}0110\rangle$																													
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}110\rangle$																				1				1					
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}110\rangle$																				1					1				
$\|10\bar{1}001\rangle\ \|\bar{1}0101\bar{1}\rangle$															1			1		1				1					
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}101\bar{1}\rangle$													1		1					1					1				
$\|10000\bar{1}\rangle\ \|\bar{1}00011\rangle$															1			1											
$\|0\bar{1}0001\rangle\ \|01001\bar{1}\rangle$													1																
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0011\rangle$													1		1														
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}011\rangle$													1												1				
$\|00\bar{1}100\rangle\ \|001\bar{1}10\rangle$																									1				
$\|000\bar{1}10\rangle\ \|000100\rangle$					$\sqrt{2}$																								
$\|0000\bar{1}0\rangle\ \|000020\rangle$					1																								
	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{9}$	2	2	2	2	$\sqrt{3}$	2	2	4	$\sqrt{3}$	$\sqrt{8}$	2	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{8}$	$\sqrt{8}$	2	$\sqrt{24}$	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{6}$	$\sqrt{5}$	$\sqrt{3}$	$\sqrt{8}$	$\sqrt{6}$

Table 15: **CG coefficients for (000010) dominant weight in (100000)⊗(000020).** The fundamental (100000) irrep is marked as $\overline{F}$, and its highest weight state as $|\,\overline{F}\rangle$. $|\mathrm{n}\rangle$ is an abbreviation for $|000010_n\rangle$. Numbering of the degenerate states is consistent with tables 3, and 1. Each CGC should be divided by the respective number in the last row to maintain $\langle n\,|\,n\rangle = 1$.

	(100020) continued from table 14											*(000011)*																$\overline{F}$
	$\|30\rangle$	$\|31\rangle$	$\|32\rangle$	$\|33\rangle$	$\|34\rangle$	$\|35\rangle$	$\|36\rangle$	$\|37\rangle$	$\|38\rangle$	$\|39\rangle$	$\|40\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|16\rangle$	$\|\overline{F}\rangle$
$\|100000\rangle\,\|\bar{1}00010_3\rangle$																					$\sqrt{2}$			$\sqrt{8}$				$\sqrt{8}$
$\|100000\rangle\,\|\bar{1}00010_4\rangle$																	$\sqrt{2}$			$\sqrt{8}$								$-\sqrt{18}$
$\|100000\rangle\,\|\bar{1}00010_5\rangle$									1													$\sqrt{2}$						$\sqrt{32}$
$\|100000\rangle\,\|\bar{1}00010_6\rangle$		1												$\sqrt{2}$	$\sqrt{8}$											$\sqrt{2}$		$-\sqrt{2}$
$\|\bar{1}00010\rangle\,\|100000_3\rangle$																					$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$		$\sqrt{8}$			$\sqrt{8}$
$\|\bar{1}00010\rangle\,\|100000_4\rangle$							$\sqrt{2}$	1			$\sqrt{2}$		$\sqrt{2}$	$\sqrt{2}$		$\sqrt{8}$						$\sqrt{2}$						$-\sqrt{18}$
$\|\bar{1}00010\rangle\,\|100000_5\rangle$									1			$\sqrt{2}$										$-\sqrt{2}$						$\sqrt{32}$
$\|\bar{1}00010\rangle\,\|100000_6\rangle$					$\sqrt{2}$												$\sqrt{2}$	$\sqrt{8}$	$\sqrt{2}$			$\sqrt{2}$				$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$
$\|\bar{1}10000\rangle\,\|1\bar{1}0010_3\rangle$																					$\sqrt{2}$			$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$		$-\sqrt{8}$
$\|\bar{1}10000\rangle\,\|1\bar{1}0010_4\rangle$																	$\sqrt{2}$	$\sqrt{2}$		$-\sqrt{2}$								$\sqrt{18}$
$\|\bar{1}10000\rangle\,\|1\bar{1}0010_5\rangle$						1			1													$\sqrt{2}$					$\sqrt{2}$	$-\sqrt{32}$
$\|\bar{1}10000\rangle\,\|1\bar{1}0010_6\rangle$		1		1										$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$										$-\sqrt{2}$		$\sqrt{2}$
$\|1\bar{1}0010\rangle\,\|\bar{1}10000_3\rangle$					$\sqrt{2}$																$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{8}$
$\|1\bar{1}0010\rangle\,\|\bar{1}10000_4\rangle$	$\sqrt{2}$		1				$\sqrt{2}$	1					$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$						$\sqrt{2}$					$\sqrt{2}$	$\sqrt{18}$
$\|1\bar{1}0010\rangle\,\|\bar{1}10000_5\rangle$						1			1			$-\sqrt{2}$										$-\sqrt{2}$					$-\sqrt{2}$	$-\sqrt{32}$
$\|1\bar{1}0010\rangle\,\|\bar{1}10000_6\rangle$																	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$		$\sqrt{2}$				$-\sqrt{2}$		$\sqrt{2}$
$\|0\bar{1}1000\rangle\,\|01\bar{1}010_3\rangle$																								$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$		$\sqrt{8}$
$\|0\bar{1}1000\rangle\,\|01\bar{1}010_4\rangle$																		$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$					$\sqrt{2}$			$-\sqrt{18}$
$\|0\bar{1}1000\rangle\,\|01\bar{1}010_5\rangle$						1																	$\sqrt{2}$				$\sqrt{2}$	$\sqrt{32}$
$\|0\bar{1}1000\rangle\,\|01\bar{1}010_6\rangle$		1		1										$-\sqrt{8}$	$-\sqrt{2}$	$-\sqrt{2}$					$-\sqrt{2}$			$-\sqrt{2}$		$-\sqrt{2}$		$-\sqrt{2}$
$\|01\bar{1}010\rangle\,\|0\bar{1}1000_3\rangle$					$\sqrt{2}$																			$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{8}$
$\|01\bar{1}010\rangle\,\|0\bar{1}1000_4\rangle$	$\sqrt{2}$		1							$\sqrt{2}$			$\sqrt{2}$		$\sqrt{2}$	$\sqrt{2}$							$\sqrt{2}$	$\sqrt{2}$			$\sqrt{2}$	$-\sqrt{18}$
$\|01\bar{1}010\rangle\,\|0\bar{1}1000_5\rangle$						1						$\sqrt{2}$											$-\sqrt{2}$				$-\sqrt{2}$	$\sqrt{32}$
$\|01\bar{1}010\rangle\,\|0\bar{1}1000_6\rangle$																	$-\sqrt{8}$	$-\sqrt{2}$		$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{2}$		$-\sqrt{2}$
$\|00\bar{1}101\rangle\,\|001\bar{1}1\bar{1}_3\rangle$				1											$\sqrt{2}$	$-\sqrt{2}$		$-\sqrt{2}$		$\sqrt{2}$				$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$		$-\sqrt{8}$
$\|00\bar{1}101\rangle\,\|001\bar{1}1\bar{1}_4\rangle$																$\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{2}$					$\sqrt{2}$			$\sqrt{18}$
$\|00\bar{1}101\rangle\,\|001\bar{1}1\bar{1}_5\rangle$													$\sqrt{2}$						$\sqrt{2}$				$\sqrt{2}$					$-\sqrt{32}$
$\|00\bar{1}101\rangle\,\|001\bar{1}1\bar{1}_6\rangle$														$\sqrt{2}$		$\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$				$\sqrt{2}$
$\|001\bar{1}1\bar{1}\rangle\,\|00\bar{1}101_3\rangle$			1												$-\sqrt{2}$	$\sqrt{2}$		$\sqrt{2}$		$-\sqrt{2}$				$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{8}$
$\|001\bar{1}1\bar{1}\rangle\,\|00\bar{1}101_4\rangle$																$-\sqrt{2}$			$\sqrt{2}$	$\sqrt{2}$			$\sqrt{2}$	$\sqrt{2}$				$\sqrt{18}$
$\|001\bar{1}1\bar{1}\rangle\,\|00\bar{1}101_5\rangle$										$\sqrt{2}$		$-\sqrt{2}$	$-\sqrt{2}$						$-\sqrt{2}$				$-\sqrt{2}$					$-\sqrt{32}$
$\|001\bar{1}1\bar{1}\rangle\,\|00\bar{1}101_6\rangle$								1						$-\sqrt{2}$		$-\sqrt{2}$	$\sqrt{2}$			$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$			$-\sqrt{2}$			$\sqrt{2}$
$\|000\bar{1}11\rangle\,\|00010\bar{1}_3\rangle$				1											$\sqrt{2}$	$-\sqrt{2}$		$\sqrt{2}$		$-\sqrt{2}$						$\sqrt{8}$		$\sqrt{8}$
$\|000\bar{1}11\rangle\,\|00010\bar{1}_4\rangle$																$\sqrt{2}$			$\sqrt{2}$	$\sqrt{2}$								$-\sqrt{18}$
$\|000\bar{1}11\rangle\,\|00010\bar{1}_5\rangle$												$\sqrt{2}$	$\sqrt{2}$						$-\sqrt{2}$									$\sqrt{32}$
$\|000\bar{1}11\rangle\,\|00010\bar{1}_6\rangle$														$\sqrt{2}$		$\sqrt{2}$	$\sqrt{2}$			$\sqrt{2}$	$\sqrt{8}$			$\sqrt{2}$	$\sqrt{2}$			$-\sqrt{2}$
$\|00010\bar{1}\rangle\,\|000\bar{1}11_3\rangle$			1			1									$-\sqrt{2}$	$\sqrt{2}$		$-\sqrt{2}$		$\sqrt{2}$							$-\sqrt{2}$	$\sqrt{8}$
$\|00010\bar{1}\rangle\,\|000\bar{1}11_4\rangle$																$-\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{18}$
$\|00010\bar{1}\rangle\,\|000\bar{1}11_5\rangle$										$\sqrt{2}$			$-\sqrt{2}$						$\sqrt{2}$									$\sqrt{32}$
$\|00010\bar{1}\rangle\,\|000\bar{1}11_6\rangle$								1	1					$-\sqrt{2}$		$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$		$-\sqrt{2}$						$-\sqrt{2}$
$\|\bar{1}001\bar{1}0\rangle\,\|100\bar{1}20\rangle$							1				1	1	1	-1		-1						1						-5
$\|1\bar{1}01\bar{1}0\rangle\,\|\bar{1}10\bar{1}20\rangle$	1						1					-1	-1	-1	-1							1					1	5
$\|01\bar{1}1\bar{1}0\rangle\,\|0\bar{1}1\bar{1}20\rangle$	1									1		1	1		-1								1	-1			1	-5
$\|0010\bar{1}\bar{1}\rangle\,\|00\bar{1}021\rangle$												-1							1	-1			1	-1				5
$\|0000\bar{1}1\rangle\,\|00002\bar{1}\rangle$												1							1	-1								-5
$\|010\bar{1}00\rangle\,\|0\bar{1}0110\rangle$	1				1					1			1		-1											-1	-1	5
$\|1\bar{1}1\bar{1}00\rangle\,\|\bar{1}1\bar{1}110\rangle$	1				1		1						-1	-1	-1						-1	-1	-1	-1		-1	-1	-5
$\|\bar{1}01\bar{1}00\rangle\,\|10\bar{1}110\rangle$							1				1		1	-1		-1					-1	-1	1		-1			5
$\|10\bar{1}001\rangle\,\|\bar{1}0101\bar{1}\rangle$																	-1		-1	-1	-1	1	-1	-1				5
$\|\bar{1}1\bar{1}001\rangle\,\|1\bar{1}101\bar{1}\rangle$					1												-1	-1	1		-1	1	1		-1	-1	1	-5
$\|10000\bar{1}\rangle\,\|\bar{1}00011\rangle$																	-1		-1	-1		-2						-5
$\|0\bar{1}0001\rangle\,\|01001\bar{1}\rangle$					1													-1	-2							-1	1	5
$\|\bar{1}1000\bar{1}\rangle\,\|1\bar{1}0011\rangle$																	-1	-1	1			-2					-2	5
$\|0\bar{1}100\bar{1}\rangle\,\|01\bar{1}011\rangle$																		-1	-2				-2		-1		-2	-5
$\|00\bar{1}100\rangle\,\|001\bar{1}10\rangle$											1		-2			-1							-2		-1			5
$\|000\bar{1}10\rangle\,\|000100\rangle$											1	-2	-2			-1												-5
$\|0000\bar{1}0\rangle\,\|000020\rangle$												$-\sqrt{2}$																$\sqrt{50}$
	$\sqrt{8}$	$\sqrt{3}$	2	2	$\sqrt{10}$	$\sqrt{5}$	$\sqrt{8}$	2	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{21}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{28}$	$\sqrt{650}$

Table 16: **CG coefficients for (001000) dominant weight in (100000)⊗(010000).** Each entry should be divided by the respective number in the last row to keep the states normalized to 1.

	(110000)		*(001000)*
	$\vert 001000_1\rangle$	$\vert 001000_2\rangle$	$\vert 001000\rangle$
$\vert 0\bar{1}1000\rangle\ \vert 010000\rangle$		1	1
$\vert \bar{1}10000\rangle\ \vert 1\bar{1}1000\rangle$	1	1	-1
$\vert 100000\rangle\ \vert \bar{1}01000\rangle$	1		1
	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{3}$

Table 17: **CG coefficients for (100010) dominant weight in (100000)⊗(010000).** $\vert n\rangle$ is an abbreviation for $\vert 100010_n\rangle$. Numbering of the degenerate states is consistent with table 4, and table II in [6]. Each CGC should be divided by the respective number in the last row to maintain $\langle n\vert n\rangle = 1$.

	(110000)								*(001000)*				*(100010)*
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 100010\rangle$
$\vert 100000\rangle\ \vert 000010_1\rangle$	$\sqrt{2}$												-2
$\vert 100000\rangle\ \vert 000010_2\rangle$		$\sqrt{2}$										$\sqrt{2}$	2
$\vert 100000\rangle\ \vert 000010_3\rangle$							$\sqrt{2}$			$\sqrt{2}$			-2
$\vert 100000\rangle\ \vert 000010_4\rangle$						$\sqrt{2}$					$\sqrt{2}$		1
$\vert 100000\rangle\ \vert 000010_6\rangle$					$\sqrt{2}$				$\sqrt{2}$				1
$\vert \bar{1}10000\rangle\ \vert 2\bar{1}0010\rangle$	1	1	1									-1	$\sqrt{2}$
$\vert 0\bar{1}1000\rangle\ \vert 11\bar{1}010\rangle$		1	1				1	1		-1		-1	-$\sqrt{2}$
$\vert 00\bar{1}101\rangle\ \vert 101\bar{1}1\bar{1}\rangle$				1	1	1	1	1	-1	-1	-1		$\sqrt{2}$
$\vert 000\bar{1}11\rangle\ \vert 10010\bar{1}\rangle$				1		1			1		-1		-$\sqrt{2}$
$\vert 00010\bar{1}\rangle\ \vert 100\bar{1}11\rangle$				1	1				-1		1		-$\sqrt{2}$
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 10\bar{1}101\rangle$				1				1	1	1	1		$\sqrt{2}$
$\vert 01\bar{1}010\rangle\ \vert 1\bar{1}1000\rangle$			1					1		1		1	-$\sqrt{2}$
$\vert 1\bar{1}0010\rangle\ \vert 010000\rangle$			1									1	$\sqrt{2}$
	$\sqrt{3}$	2	2	2	2	2	2	2	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{18}$

Table 18: **CG coefficients for (000001) dominant weight states of the 5824-dimensional (110000) irrep in (100000)⊗(010000).** $\vert\mathrm{n}\rangle$ is an abbreviation for $\vert 000001_n\rangle$. Numbering of the degenerate states is consistent with table 4. Each CGC should be divided by the respective number in the last row to maintain $\langle n \vert n\rangle = 1$.

	(110000)																							
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$	$\vert 21\rangle$	$\vert 22\rangle$	$\vert 23\rangle$	$\vert 24\rangle$
$\vert 100000\rangle \vert \bar{1}00001_1\rangle$																					1			
$\vert 100000\rangle \vert \bar{1}00001_2\rangle$																	1							
$\vert 100000\rangle \vert \bar{1}00001_3\rangle$										1														
$\vert 100000\rangle \vert \bar{1}00001_4\rangle$								1																
$\vert 100000\rangle \vert \bar{1}00001_6\rangle$																								$\sqrt{2}$
$\vert \bar{1}10000\rangle \vert 1\bar{1}0001_1\rangle$																					1	$\sqrt{2}$		
$\vert \bar{1}10000\rangle \vert 1\bar{1}0001_2\rangle$																	1		1					
$\vert \bar{1}10000\rangle \vert 1\bar{1}0001_3\rangle$										1		1												
$\vert \bar{1}10000\rangle \vert 1\bar{1}0001_4\rangle$						1		1																
$\vert \bar{1}10000\rangle \vert 1\bar{1}0001_6\rangle$																							$\sqrt{2}$	$\sqrt{2}$
$\vert 0\bar{1}1000\rangle \vert 01\bar{1}001_1\rangle$																	1	$\sqrt{2}$			1	$\sqrt{2}$		
$\vert 0\bar{1}1000\rangle \vert 01\bar{1}001_2\rangle$																			1	$\sqrt{2}$				
$\vert 0\bar{1}1000\rangle \vert 01\bar{1}001_3\rangle$												1		1										
$\vert 0\bar{1}1000\rangle \vert 01\bar{1}001_4\rangle$				1		1																		
$\vert 0\bar{1}1000\rangle \vert 01\bar{1}001_6\rangle$																$\sqrt{2}$							$\sqrt{2}$	
$\vert 00\bar{1}101\rangle \vert 001\bar{1}00_1\rangle$										1	$\sqrt{2}$						1	$\sqrt{2}$						
$\vert 00\bar{1}101\rangle \vert 001\bar{1}00_2\rangle$												1	$\sqrt{2}$						1	$\sqrt{2}$		$\sqrt{2}$		
$\vert 00\bar{1}101\rangle \vert 001\bar{1}00_3\rangle$														1	$\sqrt{2}$									
$\vert 00\bar{1}101\rangle \vert 001\bar{1}00_4\rangle$				1					1															
$\vert 00\bar{1}101\rangle \vert 001\bar{1}00_6\rangle$		$\sqrt{2}$														$\sqrt{2}$								
$\vert 000\bar{1}11\rangle \vert 0001\bar{1}0_1\rangle$							$\sqrt{2}$	1		1	$\sqrt{2}$													$\sqrt{2}$
$\vert 000\bar{1}11\rangle \vert 0001\bar{1}0_2\rangle$					$\sqrt{2}$	1						1	$\sqrt{2}$										$\sqrt{2}$	
$\vert 000\bar{1}11\rangle \vert 0001\bar{1}0_3\rangle$			$\sqrt{2}$	1										1	$\sqrt{2}$	$\sqrt{2}$				$\sqrt{2}$				
$\vert 000\bar{1}11\rangle \vert 0001\bar{1}0_4\rangle$									1															
$\vert 000\bar{1}11\rangle \vert 0001\bar{1}0_6\rangle$	$\sqrt{2}$	$\sqrt{2}$																						
$\vert 0000\bar{1}1\rangle \vert 000010_1\rangle$							$\sqrt{2}$	1																
$\vert 0000\bar{1}1\rangle \vert 000010_2\rangle$					$\sqrt{2}$	1																		
$\vert 0000\bar{1}1\rangle \vert 000010_3\rangle$			$\sqrt{2}$	1																				
$\vert 0000\bar{1}1\rangle \vert 000010_4\rangle$									1						$\sqrt{2}$									
$\vert 0000\bar{1}1\rangle \vert 000010_6\rangle$	$\sqrt{2}$																							
$\vert 0010\bar{1}\bar{1}\rangle \vert 00\bar{1}012\rangle$	1		1																	1				
$\vert 001\bar{1}1\bar{1}\rangle \vert 00\bar{1}1\bar{1}2\rangle$	1	1	1												1	1								
$\vert 00010\bar{1}\rangle \vert 000\bar{1}02\rangle$		1													1									
$\vert 01\bar{1}010\rangle \vert 0\bar{1}10\bar{1}1\rangle$			1		1											1							1	
$\vert 1\bar{1}0010\rangle \vert \bar{1}100\bar{1}1\rangle$					1		1																1	1
$\vert 01\bar{1}1\bar{1}0\rangle \vert 0\bar{1}1\bar{1}11\rangle$			1		1								1		1					1				
$\vert 010\bar{1}00\rangle \vert 0\bar{1}0101\rangle$													1		1							1		
$\vert 1\bar{1}01\bar{1}0\rangle \vert \bar{1}10\bar{1}11\rangle$					1		1				1		1											
$\vert \bar{1}00010\rangle \vert 1000\bar{1}1\rangle$							1																	1
$\vert 1\bar{1}1\bar{1}00\rangle \vert \bar{1}1\bar{1}101\rangle$											1		1					1		1		1		
$\vert \bar{1}001\bar{1}0\rangle \vert 100\bar{1}11\rangle$							1				1													
$\vert 10\bar{1}001\rangle \vert \bar{1}01000\rangle$																		1		1				
$\vert \bar{1}01\bar{1}00\rangle \vert 10\bar{1}101\rangle$											1							1						
$\vert \bar{1}1\bar{1}001\rangle \vert 1\bar{1}1000\rangle$																		1				1		
$\vert 0\bar{1}0001\rangle \vert 010000\rangle$																						1		
	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{8}$	2	$\sqrt{8}$	2	$\sqrt{8}$	2	$\sqrt{3}$	2	$\sqrt{8}$	2	$\sqrt{8}$	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{8}$	2	$\sqrt{8}$	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{3}$	$\sqrt{10}$	$\sqrt{8}$	$\sqrt{8}$

Table 19: **CG coefficients for (000001) dominant weight in (100000)⊗(010000).** The adjoint (000001) irrep is marked as A, and its highest weight state as $\lvert A\rangle$. $\lvert n\rangle$ is an abbreviation for $\lvert 000001_n\rangle$. Numbering of the degenerate states is consistent with table II in [6], and table I in [5]. Each CGC should be divided by the respective number in the last row to maintain $\langle n\vert n\rangle = 1$.

	(001000)															*(100010)*					A
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert 7\rangle$	$\lvert 8\rangle$	$\lvert 9\rangle$	$\lvert 10\rangle$	$\lvert 11\rangle$	$\lvert 12\rangle$	$\lvert 13\rangle$	$\lvert 14\rangle$	$\lvert 15\rangle$	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert A\rangle$
$\lvert 100000\rangle \lvert \bar{1}00001_1\rangle$											$\sqrt{2}$					-2	-2				-2
$\lvert 100000\rangle \lvert \bar{1}00001_2\rangle$			$\sqrt{2}$													2		-2			4
$\lvert 100000\rangle \lvert \bar{1}00001_3\rangle$												$\sqrt{2}$				-2			-2		-6
$\lvert 100000\rangle \lvert \bar{1}00001_4\rangle$							$\sqrt{2}$									1				-2	3
$\lvert 100000\rangle \lvert \bar{1}00001_6\rangle$															$\sqrt{2}$	1					5
$\lvert \bar{1}10000\rangle \lvert 1\bar{1}0001_1\rangle$											$-\sqrt{2}$					-2					2
$\lvert \bar{1}10000\rangle \lvert 1\bar{1}0001_2\rangle$			$\sqrt{2}$		$\sqrt{2}$						$\sqrt{2}$					2	2	2			-4
$\lvert \bar{1}10000\rangle \lvert 1\bar{1}0001_3\rangle$									$\sqrt{2}$			$\sqrt{2}$				-2	-2		2		6
$\lvert \bar{1}10000\rangle \lvert 1\bar{1}0001_4\rangle$						$\sqrt{2}$	$\sqrt{2}$									1	1			2	-3
$\lvert \bar{1}10000\rangle \lvert 1\bar{1}0001_6\rangle$										$\sqrt{2}$					$\sqrt{2}$	1	1				-5
$\lvert 0\bar{1}1000\rangle \lvert 01\bar{1}001_1\rangle$			$-\sqrt{2}$								$-\sqrt{2}$					2					-2
$\lvert 0\bar{1}1000\rangle \lvert 01\bar{1}001_2\rangle$					$-\sqrt{2}$						$\sqrt{2}$						2				4
$\lvert 0\bar{1}1000\rangle \lvert 01\bar{1}001_3\rangle$					$\sqrt{2}$				$\sqrt{2}$				$\sqrt{2}$	$\sqrt{2}$			-2	-2	-2		-6
$\lvert 0\bar{1}1000\rangle \lvert 01\bar{1}001_4\rangle$				$\sqrt{2}$		$\sqrt{2}$		$\sqrt{2}$									1	1		-2	3
$\lvert 0\bar{1}1000\rangle \lvert 01\bar{1}001_6\rangle$		$\sqrt{2}$								$\sqrt{2}$							1	1			5
$\lvert 00\bar{1}101\rangle \lvert 001\bar{1}00_1\rangle$			$-\sqrt{2}$									$-\sqrt{2}$				-2					2
$\lvert 00\bar{1}101\rangle \lvert 001\bar{1}00_2\rangle$					$-\sqrt{2}$				$-\sqrt{2}$		$-\sqrt{2}$						-2				-4
$\lvert 00\bar{1}101\rangle \lvert 001\bar{1}00_3\rangle$					$\sqrt{2}$								$-\sqrt{2}$	$-\sqrt{2}$				-2			6
$\lvert 00\bar{1}101\rangle \lvert 001\bar{1}00_4\rangle$				$\sqrt{2}$				$-\sqrt{2}$						$\sqrt{2}$				1	1	2	-3
$\lvert 00\bar{1}101\rangle \lvert 001\bar{1}00_6\rangle$	$\sqrt{2}$	$\sqrt{2}$											$\sqrt{2}$					1	1		-5
$\lvert 000\bar{1}11\rangle \lvert 0001\bar{1}0_1\rangle$							$-\sqrt{2}$					$-\sqrt{2}$			$-\sqrt{2}$	2					-2
$\lvert 000\bar{1}11\rangle \lvert 0001\bar{1}0_2\rangle$						$-\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{2}$							2				4
$\lvert 000\bar{1}11\rangle \lvert 0001\bar{1}0_3\rangle$		$-\sqrt{2}$		$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$					$-\sqrt{2}$	$-\sqrt{2}$				2			-6
$\lvert 000\bar{1}11\rangle \lvert 0001\bar{1}0_4\rangle$								$\sqrt{2}$						$\sqrt{2}$					1	-1	3
$\lvert 000\bar{1}11\rangle \lvert 0001\bar{1}0_6\rangle$	$-\sqrt{2}$							$\sqrt{2}$					$\sqrt{2}$						1	1	5
$\lvert 0000\bar{1}1\rangle \lvert 000010_1\rangle$							$-\sqrt{2}$								$\sqrt{2}$	-2					2
$\lvert 0000\bar{1}1\rangle \lvert 000010_2\rangle$						$-\sqrt{2}$				$\sqrt{2}$							-2				-4
$\lvert 0000\bar{1}1\rangle \lvert 000010_3\rangle$		$\sqrt{2}$		$-\sqrt{2}$				$-\sqrt{2}$										-2			6
$\lvert 0000\bar{1}1\rangle \lvert 000010_4\rangle$								$\sqrt{2}$					$\sqrt{2}$	$-\sqrt{2}$					-2	-1	-3
$\lvert 0000\bar{1}1\rangle \lvert 000010_6\rangle$	$\sqrt{2}$							$\sqrt{2}$												1	-5
$\lvert 0010\bar{1}\bar{1}\rangle \lvert 00\bar{1}012\rangle$	1	1		1	1			-1										$\sqrt{2}$		$-\sqrt{2}$	$\sqrt{8}$
$\lvert 001\bar{1}1\bar{1}\rangle \lvert 00\bar{1}1\bar{1}2\rangle$	-1	-1		1				-1					-1	1				$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{8}$
$\lvert 00010\bar{1}\rangle \lvert 000\bar{1}02\rangle$	1												-1	1					$-\sqrt{2}$		$\sqrt{8}$
$\lvert 01\bar{1}010\rangle \lvert 0\bar{1}10\bar{1}1\rangle$		-1		1		1		1		-1							$-\sqrt{2}$	$-\sqrt{2}$		$\sqrt{2}$	$\sqrt{8}$
$\lvert 1\bar{1}0010\rangle \lvert \bar{1}100\bar{1}1\rangle$						1	1			-1					-1	$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{2}$	$-\sqrt{8}$
$\lvert 01\bar{1}1\bar{1}0\rangle \lvert 0\bar{1}1\bar{1}11\rangle$		1		1	1	1		1	1	1			1	1			$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{8}$
$\lvert 010\bar{1}00\rangle \lvert 0\bar{1}0101\rangle$									1		1		1	1			$-\sqrt{2}$		$\sqrt{2}$		$\sqrt{8}$
$\lvert 1\bar{1}01\bar{1}0\rangle \lvert \bar{1}10\bar{1}11\rangle$						1	1		1	1		1			1	$\sqrt{2}$	$\sqrt{2}$		$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{8}$
$\lvert \bar{1}00010\rangle \lvert 1000\bar{1}1\rangle$							1								-1	$-\sqrt{2}$				$\sqrt{2}$	$\sqrt{8}$
$\lvert 1\bar{1}1\bar{1}00\rangle \lvert \bar{1}1\bar{1}101\rangle$			1		1				1		1	1				$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$		$-\sqrt{8}$
$\lvert \bar{1}001\bar{1}0\rangle \lvert 100\bar{1}11\rangle$							1					1			1	$\sqrt{2}$			$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{8}$
$\lvert 10\bar{1}001\rangle \lvert \bar{1}01000\rangle$			1		1											$\sqrt{2}$		$-\sqrt{2}$			$\sqrt{8}$
$\lvert \bar{1}01\bar{1}00\rangle \lvert 10\bar{1}101\rangle$			1									1				$-\sqrt{2}$		$\sqrt{2}$	$\sqrt{2}$		$\sqrt{8}$
$\lvert \bar{1}1\bar{1}001\rangle \lvert 1\bar{1}1000\rangle$			1								1					$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$			$-\sqrt{8}$
$\lvert 0\bar{1}0001\rangle \lvert 010000\rangle$											1						$\sqrt{2}$				$\sqrt{8}$
	3	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	6	6	6	6	6	$\sqrt{180}$

Table 20: **CG coefficients for the first 32 (000000) dominant weight states of the 5824-dimensional (110000) irrep in the product (100000)⊗(010000).** (The remaining 32 states of this irrep with the same weight are shown in table 21.) $|\mathrm{n}\rangle$ is an abbreviation for $|000000_n\rangle$. Numbering of the degenerate states is consistent with table 4. Each CGC should be divided by the respective number in the last row of the table to maintain $\langle n|n\rangle = 1$.

	(110000)																															
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$	$\vert 21\rangle$	$\vert 22\rangle$	$\vert 23\rangle$	$\vert 24\rangle$	$\vert 25\rangle$	$\vert 26\rangle$	$\vert 27\rangle$	$\vert 28\rangle$	$\vert 29\rangle$	$\vert 30\rangle$	$\vert 31\rangle$	$\vert 32\rangle$
$\vert 100000\rangle\,\vert\bar{1}00000_1\rangle$																																
$\vert 100000\rangle\,\vert\bar{1}00000_2\rangle$																																
$\vert 100000\rangle\,\vert\bar{1}00000_3\rangle$																																
$\vert 100000\rangle\,\vert\bar{1}00000_4\rangle$																						1										
$\vert 100000\rangle\,\vert\bar{1}00000_6\rangle$																																
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0000_1\rangle$																													1			
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0000_2\rangle$																																
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0000_3\rangle$																																
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0000_4\rangle$														1								1										
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0000_6\rangle$																																
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}000_1\rangle$																													1			
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}000_2\rangle$																																
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}000_3\rangle$																																
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}000_4\rangle$														1										1								
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}000_6\rangle$																																
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}0\bar{1}_1\rangle$							1																									
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}0\bar{1}_2\rangle$																	1															
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}0\bar{1}_3\rangle$									1					1																		
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}0\bar{1}_4\rangle$																			1					1								
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}0\bar{1}_6\rangle$															1							1										
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}\bar{1}_1\rangle$							1	1																								
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}\bar{1}_2\rangle$					1		5	2									1															
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}\bar{1}_3\rangle$	1								1																				11	5		
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}\bar{1}_4\rangle$						1													1													
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}\bar{1}_6\rangle$			1												1																	
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}01_1\rangle$							1																									
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}01_2\rangle$																	1															
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}01_3\rangle$									1					1																		
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}01_4\rangle$																			1					1								
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}01_6\rangle$															1							1										
$\vert 0000\bar{1}1\rangle\,\vert 00001\bar{1}_1\rangle$							1	1									1	1														
$\vert 0000\bar{1}1\rangle\,\vert 00001\bar{1}_2\rangle$					1		5	2															1									
$\vert 0000\bar{1}1\rangle\,\vert 00001\bar{1}_3\rangle$	1											1																	11	5		
$\vert 0000\bar{1}1\rangle\,\vert 00001\bar{1}_4\rangle$						1																										
$\vert 0000\bar{1}1\rangle\,\vert 00001\bar{1}_6\rangle$			1																	1												
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}1_1\rangle$							1	1																								
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}1_2\rangle$					1		5	2									1															
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}1_3\rangle$	1								1																1	1		1	10	5		
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}1_4\rangle$						1													1													
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}1_6\rangle$			1												1																1	
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}011_1\rangle$							1	1									1	1														
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}011_2\rangle$					1		5	2															1									
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}011_3\rangle$	1											1													1	1		1	10	5		
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}011_4\rangle$						1																										
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}011_6\rangle$			1																	1											1	
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}0_1\rangle$																													1	1		
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}0_2\rangle$	1	1			1		5	3																					10	5		
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}0_3\rangle$																									1	1	1	1				
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}0_4\rangle$																												1				
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}0_6\rangle$																										1					1	
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}10_1\rangle$																													1	1		
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}10_2\rangle$	1	1			1		5	3	1	1		1					1	1					1						10	5		
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}10_3\rangle$																									1	1	1	1				
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}10_4\rangle$																												1				
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}10_6\rangle$																										1					1	
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}0_1\rangle$																													1	1		
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}0_2\rangle$	1	1	1	1		1																							10	5		
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}0_3\rangle$																									1	1	1	1			1	1
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}0_4\rangle$																												1				
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}0_6\rangle$																										1						
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0100_1\rangle$																																
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0100_2\rangle$									1	1			1	1			1	1														
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0100_3\rangle$																																
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0100_4\rangle$																											1	1				
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0100_6\rangle$																																

continued on next page

continued from previous page

	(110000)																															
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert 7\rangle$	$\lvert 8\rangle$	$\lvert 9\rangle$	$\lvert 10\rangle$	$\lvert 11\rangle$	$\lvert 12\rangle$	$\lvert 13\rangle$	$\lvert 14\rangle$	$\lvert 15\rangle$	$\lvert 16\rangle$	$\lvert 17\rangle$	$\lvert 18\rangle$	$\lvert 19\rangle$	$\lvert 20\rangle$	$\lvert 21\rangle$	$\lvert 22\rangle$	$\lvert 23\rangle$	$\lvert 24\rangle$	$\lvert 25\rangle$	$\lvert 26\rangle$	$\lvert 27\rangle$	$\lvert 28\rangle$	$\lvert 29\rangle$	$\lvert 30\rangle$	$\lvert 31\rangle$	$\lvert 32\rangle$
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_1\rangle$																													1	1		
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_2\rangle$	1	1	1	1		1			1	1		1			1	1			1	1									10	5		
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_3\rangle$																									1	1	1	1			1	1
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_4\rangle$																												1				
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_6\rangle$																										1						
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_1\rangle$																																
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_2\rangle$			1	1																												
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_3\rangle$																															1	1
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_4\rangle$																																
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_6\rangle$																										1	1					
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_1\rangle$																																
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_2\rangle$									1	1			1	1	1	1			1		1	1		1								
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_3\rangle$																																
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_4\rangle$																											1	1				1
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_6\rangle$																																
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_1\rangle$																																
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_2\rangle$			1	1											1	1				1												
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_3\rangle$																															1	1
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_4\rangle$																																
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_6\rangle$																										1	1					
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_1\rangle$											1		1	2								1										
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_2\rangle$																					1			1								
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_3\rangle$																1			1													
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_4\rangle$				1		1																										1
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_6\rangle$																				1												
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_1\rangle$																																
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_2\rangle$															1	1					1	1										
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_3\rangle$																																
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_4\rangle$																																1
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_6\rangle$																																
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_1\rangle$											1		1	2								1										
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_2\rangle$																					1			1								
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_3\rangle$																1			1													
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_4\rangle$				1		1																										
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_6\rangle$																				1												
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_1\rangle$											1			1								1										
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_2\rangle$													1								1											
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_3\rangle$										1						1																
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_4\rangle$		1		1																							1					1
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_6\rangle$												1								1												
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_1\rangle$											1			1								1										
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_2\rangle$													1								1											
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_3\rangle$										1						1																
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_4\rangle$		1		1																										1		
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_6\rangle$												1								1												
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_1\rangle$											1			1																		
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_2\rangle$													1																			
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_3\rangle$										1								1														
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_4\rangle$		1						1																			1					
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_6\rangle$												1											1									
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_1\rangle$											1			1																		
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_2\rangle$													1																			
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_3\rangle$										1								1														
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_4\rangle$		1						1																						1		
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_6\rangle$												1											1									
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_1\rangle$																																
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_2\rangle$																																
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_3\rangle$																																
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_4\rangle$																																
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_6\rangle$																		1					1									
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_1\rangle$																																
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_2\rangle$																														1		
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_3\rangle$																																
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_4\rangle$																																
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_6\rangle$								1										1														
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_1\rangle$																																
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_2\rangle$																														1		
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_3\rangle$																																
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_4\rangle$																																
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_6\rangle$								1																								
	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{8}$	$\sqrt{176}$	$\sqrt{50}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{18}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{10}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	4	$\sqrt{10}$	4	$\sqrt{888}$	$\sqrt{228}$	$\sqrt{8}$	$\sqrt{8}$

Table 21: **CG coefficients for the remaining 32 (000000) dominant weight states of the 5824-dimensional (110000) irrep in the product (100000)⊗(010000).** (The first 32 states of this irrep with the same weight are shown in table 20.) |n⟩ is an abbreviation for $\vert 000000_n\rangle$. Numbering of the degenerate states is consistent with table 4. Each CGC should be divided by the respective number in the last row to maintain $\langle n\vert n\rangle = 1$.

	(110000)																															
	|33⟩	|34⟩	|35⟩	|36⟩	|37⟩	|38⟩	|39⟩	|40⟩	|41⟩	|42⟩	|43⟩	|44⟩	|45⟩	|46⟩	|47⟩	|48⟩	|49⟩	|50⟩	|51⟩	|52⟩	|53⟩	|54⟩	|55⟩	|56⟩	|57⟩	|58⟩	|59⟩	|60⟩	|61⟩	|62⟩	|63⟩	|64⟩
$\vert 100000\rangle\ \vert\bar{1}00000_1\rangle$			1																													
$\vert 100000\rangle\ \vert\bar{1}00000_2\rangle$																						1										
$\vert 100000\rangle\ \vert\bar{1}00000_3\rangle$																													1			
$\vert 100000\rangle\ \vert\bar{1}00000_4\rangle$																																
$\vert 100000\rangle\ \vert\bar{1}00000_6\rangle$																	1															
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_1\rangle$			1																													
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_2\rangle$												1										1										
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_3\rangle$															1														1			
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_4\rangle$																																
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_6\rangle$																	1	1														
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_1\rangle$						1																										
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_2\rangle$												1											1									
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_3\rangle$															1																	1
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_4\rangle$																																
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_6\rangle$																	1	1									1		1			
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_1\rangle$						1																										
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_2\rangle$																							1									
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_3\rangle$												1			1			1														1
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_4\rangle$																																
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_6\rangle$																						1					1		1			
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_1\rangle$																																
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_2\rangle$						11	5																1									1
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_3\rangle$												1																				
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_4\rangle$																																
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_6\rangle$			1																			1										
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_1\rangle$								1																								
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_2\rangle$																										1						
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_3\rangle$											1																					
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_4\rangle$																									1							1
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_6\rangle$																				1												
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_1\rangle$																																
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_2\rangle$						11	5																									
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_3\rangle$																																
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_4\rangle$																																
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_6\rangle$			1																													
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_1\rangle$						1	1	1																								
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_2\rangle$				1		10	5																			1						
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_3\rangle$											1							1														1
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_4\rangle$					1																				1							
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_6\rangle$																				1							1					
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_1\rangle$						1	1	1															1	1		1						
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_2\rangle$				1		10	5																								1	1
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_3\rangle$														1																		
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_4\rangle$					1																											
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_6\rangle$																														1		
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_1\rangle$						1	1																									
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_2\rangle$				1		10	5																									
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_3\rangle$																																1
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_4\rangle$					1					1	1														2							
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_6\rangle$																		1									1					
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_1\rangle$						1	1					1	1										1	1								
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_2\rangle$				1		10	5																			1					1	1
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_3\rangle$										1	1			1				1														
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_4\rangle$					1					1															1							
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_6\rangle$									1		1									1										1		
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_1\rangle$		1	1																													
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_2\rangle$																																
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_3\rangle$				1	1																									1		
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_4\rangle$	1									1	1								1	1					1	1						
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_6\rangle$																		1														
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_1\rangle$												1	1		1	1							1	1							1	2
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_2\rangle$																		1								1						
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_3\rangle$										1	1																					
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_4\rangle$										1															1							
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_6\rangle$									1		1									1												

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continued from previous page

	(110000)																														
	$\|33\rangle$	$\|34\rangle$	$\|35\rangle$	$\|36\rangle$	$\|37\rangle$	$\|38\rangle$	$\|39\rangle$	$\|40\rangle$	$\|41\rangle$	$\|42\rangle$	$\|43\rangle$	$\|44\rangle$	$\|45\rangle$	$\|46\rangle$	$\|47\rangle$	$\|48\rangle$	$\|49\rangle$	$\|50\rangle$	$\|51\rangle$	$\|52\rangle$	$\|53\rangle$	$\|54\rangle$	$\|55\rangle$	$\|56\rangle$	$\|57\rangle$	$\|58\rangle$	$\|59\rangle$	$\|60\rangle$	$\|61\rangle$	$\|62\rangle$	$\|63\rangle$	$\|64\rangle$
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_1\rangle$		1	1									1	1								1	1										
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_2\rangle$																																
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_3\rangle$				1	1					1	1			1				1	1	1					1	1	1			4	1	1
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_4\rangle$	1									1									1						1							
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_6\rangle$									1		1																					
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_1\rangle$		1	1																													
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_2\rangle$																																
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_3\rangle$																														1		
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_4\rangle$	1																		1	1												
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_6\rangle$																																
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_1\rangle$												1	1		1	1					1	1						1	1			1
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_2\rangle$																		1									1				1	2
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_3\rangle$										1	1								1	1					1	1				2		
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_4\rangle$	1				1					1									1						1					1		
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_6\rangle$									1		1																			1		
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_1\rangle$		1	1																		1	1										
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_2\rangle$																																
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_3\rangle$																			1	1							1			4		
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_4\rangle$	1																		1							1						
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_6\rangle$									1	1	1							1														
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_1\rangle$															1	1	1	1										1	1			1
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_2\rangle$																											1				1	2
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_3\rangle$																			1						1					2		
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_4\rangle$					1																									1		
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_6\rangle$																														1		
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_1\rangle$																					1	1						1	1			
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_2\rangle$																											1					1
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_3\rangle$																			1	1										2	1	
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_4\rangle$	1			1															1							1				1		
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_6\rangle$									1	1	1			1																1		
$\|10000\bar{1}\rangle\ \|\bar{1}00001_1\rangle$																																
$\|10000\bar{1}\rangle\ \|\bar{1}00001_2\rangle$																												1				1
$\|10000\bar{1}\rangle\ \|\bar{1}00001_3\rangle$																					1											
$\|10000\bar{1}\rangle\ \|\bar{1}00001_4\rangle$		1																														
$\|10000\bar{1}\rangle\ \|\bar{1}00001_6\rangle$																																
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_1\rangle$																	1											1	1			
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_2\rangle$																		1									1					1
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_3\rangle$										1									1							1				2	1	
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_4\rangle$				1																										1		
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_6\rangle$														1																1		
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_1\rangle$																																
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_2\rangle$																1												1			1	2
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_3\rangle$													1								1											
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_4\rangle$		1																														
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_6\rangle$																																
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_1\rangle$																	1															
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_2\rangle$																		1														
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_3\rangle$										1																						
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_4\rangle$																																
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_6\rangle$														1																	1	
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_1\rangle$																												1				
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_2\rangle$																1																1
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_3\rangle$													1											1		1					1	
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_4\rangle$				1			1																									
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_6\rangle$																																
$\|00\bar{1}100\rangle\ \|001\bar{1}00_1\rangle$																					1							1				
$\|00\bar{1}100\rangle\ \|001\bar{1}00_2\rangle$													1			1																1
$\|00\bar{1}100\rangle\ \|001\bar{1}00_3\rangle$																								1							1	
$\|00\bar{1}100\rangle\ \|001\bar{1}00_4\rangle$				1			1	1																		1						
$\|00\bar{1}100\rangle\ \|001\bar{1}00_6\rangle$																																
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_1\rangle$		1																			1											
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_2\rangle$													1																			
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_3\rangle$							1																	1								
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_4\rangle$								1																		1						
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_6\rangle$																																
$\|0000\bar{1}0\rangle\ \|000010_1\rangle$		1																														
$\|0000\bar{1}0\rangle\ \|000010_2\rangle$																																
$\|0000\bar{1}0\rangle\ \|000010_3\rangle$							1																									
$\|0000\bar{1}0\rangle\ \|000010_4\rangle$								1																								
$\|0000\bar{1}0\rangle\ \|000010_6\rangle$																																
	$\sqrt{6}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{10}$	$\sqrt{8}$	$\sqrt{688}$	$\sqrt{178}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{14}$	4	$\sqrt{10}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{6}$	$\sqrt{6}$	4	$\sqrt{14}$	$\sqrt{10}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{76}$	$\sqrt{12}$	6

Table 22: **CG coefficients for the first 36 (000000) dominant weight states of the 2925-dimensional (001000) irrep in the product (100000)⊗(010000).** (The remaining 9 states of this irrep with the same weight are shown in table 23.) $\vert n\rangle$ is an abbreviation for $\vert 000000_n\rangle$. Numbering of the degenerate states is consistent with table II. in ref.[6]. Each CGC should be divided by the respective number in the last row to maintain $\langle n\vert n\rangle = 1$.

	(001000)																																			
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$	$\vert 21\rangle$	$\vert 22\rangle$	$\vert 23\rangle$	$\vert 24\rangle$	$\vert 25\rangle$	$\vert 26\rangle$	$\vert 27\rangle$	$\vert 28\rangle$	$\vert 29\rangle$	$\vert 30\rangle$	$\vert 31\rangle$	$\vert 32\rangle$	$\vert 33\rangle$	$\vert 34\rangle$	$\vert 35\rangle$	$\vert 36\rangle$
$\vert 100000\rangle\ \vert\bar{1}00000_1\rangle$																				1																
$\vert 100000\rangle\ \vert\bar{1}00000_2\rangle$																																				1
$\vert 100000\rangle\ \vert\bar{1}00000_3\rangle$																																				
$\vert 100000\rangle\ \vert\bar{1}00000_4\rangle$												1																								
$\vert 100000\rangle\ \vert\bar{1}00000_6\rangle$																														1			1			
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_1\rangle$																		1		1																
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_2\rangle$																													1							1
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_3\rangle$																																	1			
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_4\rangle$							1					1																								
$\vert\bar{1}10000\rangle\ \vert 1\bar{1}0000_6\rangle$																														1			1	1		
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_1\rangle$																		1						1												
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_2\rangle$																													1							
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_3\rangle$																																	1			
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_4\rangle$							1							1																						
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_6\rangle$																														1			1	1		
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_1\rangle$			1																					1		1										
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_2\rangle$									1																											
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_3\rangle$					1		1																						1				1	1		
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_4\rangle$										1				1	1																					
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_6\rangle$								1				1																								1
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_1\rangle$			-1																							1										
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_2\rangle$			1						1				1									1		1												
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_3\rangle$	1				1	1									1			1											1							
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_4\rangle$				1						1					-1																					
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_6\rangle$		1						1			1									1																1
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_1\rangle$			1																							-1										
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_2\rangle$									1																											
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_3\rangle$					1		1																				1	1								
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_4\rangle$										1				1	1																					
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_6\rangle$								1				1																							1	
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_1\rangle$			-1						-1																	1										
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_2\rangle$			1										-1									1		1												
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_3\rangle$	1					-1									-1			1																		
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_4\rangle$				1											1																					
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_6\rangle$		1									-1				1					1																
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_1\rangle$			-1																					-1		-1										
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_2\rangle$			1						1			1										-1														
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_3\rangle$	1				1	1									1	1	1					1					1	1				1		1		
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_4\rangle$				1						1					-1								1													
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_6\rangle$		1						1			1								1		1														1	
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_1\rangle$			-1						-1															-1		-1										
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_2\rangle$			1										-1									-1														
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_3\rangle$	1					-1									-1	1	1					1										-1				
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_4\rangle$				1											1								1													
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_6\rangle$		1									-1				1				1		1															
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_1\rangle$																		-1						-1												
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_2\rangle$	-1		-1			1							1		1							-1														
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_3\rangle$																-1	-1					1										1				
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_4\rangle$																1							1		1		1	-1								
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_6\rangle$																	1		1		-1										1			1		
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_1\rangle$																		-1						-1					-1							
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_2\rangle$	-1		-1		-1	-1			-1				-1		-1							-1														
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_3\rangle$																-1	-1					1					-1	-1				-1		-1		
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_4\rangle$																1							1		1			1								
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_6\rangle$																	1		1		-1							1			-1				1	
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_1\rangle$																		-1		-1																
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_2\rangle$	-1	-1		-1		1					1				-1																					
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_3\rangle$																-1	-1		-1		-1	-1	-1									1				
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_4\rangle$																1					1				-1		1	-1							-1	
$\vert 1\bar{1}0010\rangle\ \vert\bar{1}100\bar{1}0_6\rangle$																	1				1										1			1		
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_1\rangle$																													-1				-1			
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_2\rangle$					-1		-1		-1																									-1		
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_3\rangle$																											-1	-1						1		
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_4\rangle$																-1	1								1			1								
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_6\rangle$																												1			1				1	

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(001000)

	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|16\rangle$	$\|17\rangle$	$\|18\rangle$	$\|19\rangle$	$\|20\rangle$	$\|21\rangle$	$\|22\rangle$	$\|23\rangle$	$\|24\rangle$	$\|25\rangle$	$\|26\rangle$	$\|27\rangle$	$\|28\rangle$	$\|29\rangle$	$\|30\rangle$	$\|31\rangle$	$\|32\rangle$	$\|33\rangle$	$\|34\rangle$	$\|35\rangle$	$\|36\rangle$
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_1\rangle$																		-1		-1									-1							-1
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_2\rangle$	-1	-1		-1	-1	-1		-1		-1	-1																									
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_3\rangle$																-1	-1		-1		-1	-1	-1					-1	-1				-1		-1	-1	
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_4\rangle$																1					1				-1			1							1	
$\|1\bar{1}01\bar{1}0\rangle\ \|\bar{1}10\bar{1}10_6\rangle$																	1				1							1			-1					
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_1\rangle$																				-1																
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_2\rangle$		-1		1							1																									
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_3\rangle$																			-1		-1		1													
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_4\rangle$																					1				1										-1	
$\|\bar{1}00010\rangle\ \|1000\bar{1}0_6\rangle$																1	-1				1										1					
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_1\rangle$																													-1				-1			-1
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_2\rangle$					-1		-1	-1		-1		-1		-1	-1																					
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_3\rangle$																											-1	-1						1	-1	
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_4\rangle$																-1	1		1		-1		-1		-1			1							1	
$\|1\bar{1}1\bar{1}00\rangle\ \|\bar{1}1\bar{1}100_6\rangle$																												1			1					
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_1\rangle$																				-1																-1
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_2\rangle$		-1		1				-1		1	-1				-1																					
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_3\rangle$																			-1		-1		1												-1	
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_4\rangle$																					1				1										1	
$\|\bar{1}001\bar{1}0\rangle\ \|100\bar{1}10_6\rangle$																1	-1				1						1	-1			-1			-1		
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_1\rangle$							-1					-1																		-1			-2	-1		
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_2\rangle$												1		-1																						
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_3\rangle$								1		-1					-1																				1	
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_4\rangle$		1		-1											1				1		1		-1													
$\|10\bar{1}001\rangle\ \|\bar{1}0100\bar{1}_6\rangle$											1				1																					
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_1\rangle$																																				-1
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_2\rangle$								-1		1		-1		1	1																					
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_3\rangle$																																			-1	
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_4\rangle$																			1		-1	1	1		1										1	
$\|\bar{1}01\bar{1}00\rangle\ \|10\bar{1}100_6\rangle$																											1	-1			1	1		1		
$\|10000\bar{1}\rangle\ \|\bar{1}00001_1\rangle$							-1					-1																		1			1	1		
$\|10000\bar{1}\rangle\ \|\bar{1}00001_2\rangle$												1		-1																						
$\|10000\bar{1}\rangle\ \|\bar{1}00001_3\rangle$								1		-1					-1																					1
$\|10000\bar{1}\rangle\ \|\bar{1}00001_4\rangle$		1		-1											1					1																
$\|10000\bar{1}\rangle\ \|\bar{1}00001_6\rangle$											1				1																					
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_1\rangle$												-1																		-1			-1			
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_2\rangle$							1					1		1																				-1		
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_3\rangle$					1			1		1					1												1	1						1	1	
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_4\rangle$	1	1		1												1	1		1		1	1	1													
$\|\bar{1}1\bar{1}001\rangle\ \|1\bar{1}100\bar{1}_6\rangle$						1					1				-1																	1				
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_1\rangle$												-1																		1			1			
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_2\rangle$							1					1		1																			1	1		
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_3\rangle$					1			1		1					1														1							1
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_4\rangle$	1	1		1														1		1																
$\|\bar{1}1000\bar{1}\rangle\ \|1\bar{1}0001_6\rangle$						1					1				-1																					
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_1\rangle$												1																		-1			-1			
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_2\rangle$							1																											-1		
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_3\rangle$					1				1																		1	1						1		
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_4\rangle$	1		1													1	1																			
$\|0\bar{1}0001\rangle\ \|01000\bar{1}_6\rangle$						1							1		1																	1				
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_1\rangle$												1																		1			1			
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_2\rangle$							1																										1	1		
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_3\rangle$					1				1																				1							
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_4\rangle$	1		1															1				1		1												
$\|0\bar{1}100\bar{1}\rangle\ \|01\bar{1}001_6\rangle$						1							1		1																					
$\|00\bar{1}100\rangle\ \|001\bar{1}00_1\rangle$																																				1
$\|00\bar{1}100\rangle\ \|001\bar{1}00_2\rangle$																													1				1			
$\|00\bar{1}100\rangle\ \|001\bar{1}00_3\rangle$																																				
$\|00\bar{1}100\rangle\ \|001\bar{1}00_4\rangle$																						1		1		1										
$\|00\bar{1}100\rangle\ \|001\bar{1}00_6\rangle$									1				1																							
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_1\rangle$																				1																1
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_2\rangle$																		1											1							
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_3\rangle$																								1												
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_4\rangle$																										1										
$\|000\bar{1}10\rangle\ \|0001\bar{1}0_6\rangle$			1						1																											
$\|0000\bar{1}0\rangle\ \|000010_1\rangle$																				1																
$\|0000\bar{1}0\rangle\ \|000010_2\rangle$																		1																		
$\|0000\bar{1}0\rangle\ \|000010_3\rangle$																								1												
$\|0000\bar{1}0\rangle\ \|000010_4\rangle$																										1										
$\|0000\bar{1}0\rangle\ \|000010_6\rangle$			1																																	
	$\sqrt{12}$ each of the first twelve states												3	3	$\sqrt{18}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	3	3	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	3	3	3	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{12}$	$\sqrt{12}$

Table 23: **CG coefficients for the (000000) dominant weight states of the 2925-dimensional (001000) irrep, 650-dimensional (100010) irrep, and 78-dimensional (000001) irrep in the product (100000)⊗(010000).** $|n\rangle$ is an abbreviation for $|000000_n\rangle$. Numbering of the degenerate states is consistent with table II. in ref.[6] and table I in ref.[5]. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(001000)									*(100010)*																				*(000001)*					
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|16\rangle$	$\|17\rangle$	$\|18\rangle$	$\|19\rangle$	$\|20\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$
$\|100000\rangle\ \|\bar{1}00000_1\rangle$										-2											-2												4		-2
$\|100000\rangle\ \|\bar{1}00000_2\rangle$										2	-2																					4			4
$\|100000\rangle\ \|\bar{1}00000_3\rangle$						1				-2								-2													4				-6
$\|100000\rangle\ \|\bar{1}00000_4\rangle$										1																		-2		4					3
$\|100000\rangle\ \|\bar{1}00000_6\rangle$										1			-2																					4	5
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_1\rangle$										-2		-2									-2	-2		-2									-4	-2	-2
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_2\rangle$										2	-2	2		-2						-2												-4		4	4
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_3\rangle$						1				-2		-2	-2			-2	-2	-2													-4			-6	-6
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_4\rangle$										1		1															-2	-2	-2	-4				3	3
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_6\rangle$										1		1	2																					1	5
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_1\rangle$												-2			-2							-2	-2	2							-2		4	-2	
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_2\rangle$		1										2		-2	2		-2		-2	2											4	4		4	
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_3\rangle$								1				-2	-2		-2	2	2														-2			-6	
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_4\rangle$												1			1	-2											2		-2	4	3			3	
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_6\rangle$					1	1						1	2		1			2													1			1	-4
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_1\rangle$															-2				-2				-2	-2	-2	-2				-2	-2	-2	-4		
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_2\rangle$		1		1											2		-2		2	-2					2		-2			4	4				
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_3\rangle$								1					2	2	-2	2	2			2					-2		2		2	-2	-2	-2		4	
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_4\rangle$															1	-2				-2					1		-2			-1	3	3			
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_6\rangle$					1	1					2				1			2							1			2		1	1	1			4
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_1\rangle$																			-2					2	-2					2		-2	2		
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_2\rangle$		1		1				1	1							-2	2		2	-2			2	-2	2	2	2			-4	-4				
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_3\rangle$														2						2		2		2	-2	-2	-2		-2	2		-2	-2	-4	
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_4\rangle$																				-2				-2	1	1	2			1		3	3		
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_6\rangle$											2										2				1	1		-2		-1		1	1		-4
$\|00010\bar{1}\rangle\ \|000\bar{1}01_1\rangle$																								2	-2	-2				-2		2	4		
$\|00010\bar{1}\rangle\ \|000\bar{1}01_2\rangle$				-1																2					2		-2			4					
$\|00010\bar{1}\rangle\ \|000\bar{1}01_3\rangle$				1										-2					-2	-2					-2		2		2	-2		2		-4	
$\|00010\bar{1}\rangle\ \|000\bar{1}01_4\rangle$			1					1	1							2	-2		1	2					1		-2			-1	-4	-3			
$\|00010\bar{1}\rangle\ \|000\bar{1}01_6\rangle$	1										-2								1						1			2		1		-1			-4
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_1\rangle$				1															2	2				2	2					-2		4	2		
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_2\rangle$									-1						2	2							2	-2		2				4					
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_3\rangle$												2										2		2		-2			2	-2			-2	4	
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_4\rangle$																								-2		1	-1			-1	4		3		
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_6\rangle$										2											2					1	1	2		1			1		4
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_1\rangle$															-2									-2	-2					2	2	2	-2		
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_2\rangle$				-1					1						2	2				2				2	2	2	2			-4					
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_3\rangle$				1				1	1				-2	-2	-2	-2	-2		-2	-2			-2	-2	-2	-2	-2		-2	2	2	2	2	4	
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_4\rangle$			1						-1						1		2		1	2		-2	1	2	1	1	2			1	1	-3	-3		
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_6\rangle$	1				1		1				-2				1			-2	1		-2		1		1	1		-2		-1	-1	-1	-1		4
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_1\rangle$		-1		-1											2					-2				-2	2					-2	-2	-4	-2		
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_2\rangle$								-1	-1						-2	-2								2		2				4					
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_3\rangle$									-1			-2	2			2						-2	-2	-2		-2			2	-2	-2		2	-4	
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_4\rangle$									1								-1						1	2		1	-1			-1	-1		-3		
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_6\rangle$							-1		1	-2							1	2			-2		1			1	1	2		1	1		-1		-4
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_1\rangle$												-2			-2									2							2		2	2	
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_2\rangle$									1			2			2	2								-2		-2	-2		2	4					
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_3\rangle$								1	1			-2	2		-2	-2	-2						-2	2							2		-2	-2	
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_4\rangle$			2	1					-1			1		2	1		2			-2		1	1	-2							1	4	3	1	
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_6\rangle$					1		1					1	-2		1			-2			2	1	1								-1		1	-1	-4
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_1\rangle$		-1										2			2					2				2							-2	2	2	-2	
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_2\rangle$				-1				-1	-1			-2			-2	-2				-2				-2	-2	-2			-2	-4					
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_3\rangle$				1					-1			2	-2			2			-2	2			-2	2							-2	-2	-2	2	
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_4\rangle$			1						1					-1			-1		1			1	1	-2							-1	-1	3	-1	
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_6\rangle$	-1						-1		1	2	2		2	1			1	2	1		2	1	1								1	1	1	1	4
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_1\rangle$										-2		-2												-2									-2	2	2
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_2\rangle$										2		2												2		2	2	2	2	-4					
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_3\rangle$							1		-1	-2		-2	2			2	2	2					2	-2							-4		2	-2	-2
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_4\rangle$	1		-1	-1						1	2	1		2					2	2	1	1		2								-4	-3	1	1
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_6\rangle$										1		1	-2								-1	1											-1	-1	3
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_1\rangle$		-1						-1				-2			-2	2				2											4	2		2	
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_2\rangle$				-1								2								-2					-2				2	4					
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_3\rangle$				1								-2	2						-2	2												-2		-2	
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_4\rangle$			1										1	-1					1			-2		2								-1	-4	1	
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_6\rangle$	-1									-2	2		-1	1					1													1		-1	-4

continued on next page

continued from previous page

Columns (001000):

	(001000)								
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert 7\rangle$	$\lvert 8\rangle$	$\lvert 9\rangle$
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{1}\rangle$									
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{2}\rangle$									
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{3}\rangle$	-1		-1	-1	-1		-1	-1	
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{4}\rangle$			-1						
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{6}\rangle$	1								
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{1}\rangle$									
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{2}\rangle$									
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{3}\rangle$							1		
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{4}\rangle$	1		1						
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{6}\rangle$									
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{1}\rangle$						-1			
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{2}\rangle$					-1			-1	
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{3}\rangle$	-1		-1	-1	1				-1
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{4}\rangle$			-1						1
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{6}\rangle$	1						1		1
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{1}\rangle$									
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{2}\rangle$									
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{3}\rangle$	-1		1		-1		-1		-1
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{4}\rangle$			1	1					
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{6}\rangle$	1								
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{1}\rangle$						-1			
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{2}\rangle$					-1			-1	
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{3}\rangle$	1		-1		1				-1
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{4}\rangle$									1
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{6}\rangle$							1		1
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{1}\rangle$						-1			
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{2}\rangle$					-1				
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{3}\rangle$	-1		1		1			1	1
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{4}\rangle$			1	1					
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{6}\rangle$	1						1		-1
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{1}\rangle$									
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{2}\rangle$					1	1			
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{3}\rangle$									
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{4}\rangle$									
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{6}\rangle$									
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{1}\rangle$						-1			
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{2}\rangle$					-1				
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{3}\rangle$	1		1	1	1			1	1
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{4}\rangle$									
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{6}\rangle$							1		-1
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{1}\rangle$									
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{2}\rangle$					1	1		1	
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{3}\rangle$									
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{4}\rangle$									
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{6}\rangle$									
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{1}\rangle$									
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{2}\rangle$									
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{3}\rangle$									
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{4}\rangle$									
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{6}\rangle$								1	2
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{1}\rangle$						1			
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{2}\rangle$									
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{3}\rangle$		1		1				1	
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{4}\rangle$									
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{6}\rangle$									1
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{1}\rangle$						1			
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{2}\rangle$									
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{3}\rangle$		1						1	
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{4}\rangle$				1					
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{6}\rangle$									1
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{1}\rangle$									
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{2}\rangle$									
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{3}\rangle$		1							
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{4}\rangle$				1					
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{6}\rangle$									
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{1}\rangle$									
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{2}\rangle$									
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{3}\rangle$									
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{4}\rangle$									
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{6}\rangle$									
	$\sqrt{12}$	3	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{18}$	$\sqrt{27}$

Columns (100010), $\lvert 1\rangle$–$\lvert 10\rangle$:

	(100010)									
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert 7\rangle$	$\lvert 8\rangle$	$\lvert 9\rangle$	$\lvert 10\rangle$
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{1}\rangle$	2		2							
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{2}\rangle$	-2		-2							
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{3}\rangle$	2		2	-2		2	-2		-2	2
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{4}\rangle$		-1			-1					-2
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{6}\rangle$	-2	-1		2	1					
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{1}\rangle$	-2									
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{2}\rangle$	2									
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{3}\rangle$	-2								2	
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{4}\rangle$	1	2								-2
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{6}\rangle$	1			2						
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{1}\rangle$	-2		-2				-2			
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{2}\rangle$	2		2			2	2			
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{3}\rangle$	-2		-2	2		-2			2	2
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{4}\rangle$		-1		-1	-1			-1	-1	-2
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{6}\rangle$	2	-1		-1	1			1	-1	
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{1}\rangle$	2									
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{2}\rangle$	-2									
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{3}\rangle$	2					-2		-2	-2	-2
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{4}\rangle$		-1								2
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{6}\rangle$	-2	-1	-2	-2	-2					
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{1}\rangle$	2		2				-2			
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{2}\rangle$	-2					2	2			
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{3}\rangle$	2	2				-2			2	-2
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{4}\rangle$								-1	-1	
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{6}\rangle$	-2							1	-1	
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{1}\rangle$	-2						2			
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{2}\rangle$	2					-2	-2			
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{3}\rangle$	-2					2	2	2	2	-2
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{4}\rangle$		-1					-1		-1	2
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{6}\rangle$	2	-1	2		-2		-1	-2	-1	
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{1}\rangle$	-2		-2	-2						
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{2}\rangle$	2					-2	-2		-2	
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{3}\rangle$	-2	-2								
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{4}\rangle$										
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{6}\rangle$	2									
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{1}\rangle$	2						2			
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{2}\rangle$	-2		-2			-2	-2			
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{3}\rangle$	2	2	2	2	2	2	2	2	2	2
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{4}\rangle$				-1			-1		-1	
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{6}\rangle$	-2		-2	-1			-1	-2	-1	
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{1}\rangle$	-2			2						
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{2}\rangle$	2		2	-2		2		-2	-2	
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{3}\rangle$	-2	-2	-2		-2					
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{4}\rangle$										
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{6}\rangle$	2		2							
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{1}\rangle$	-2									
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{2}\rangle$			-2							
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{3}\rangle$			2	2	2					
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{4}\rangle$				-1						
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{6}\rangle$			-2	-1		-2		2		
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{1}\rangle$	2			2			2		2	
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{2}\rangle$			2	-2			-2	2		
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{3}\rangle$			-2		-2	-2	2	-2		-2
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{4}\rangle$							-1			
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{6}\rangle$			2			2	-1			
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{1}\rangle$	-2	2					2		2	
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{2}\rangle$			-2	2	2		-2	2		
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{3}\rangle$						-2	2	-2		2
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{4}\rangle$							-1			-2
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{6}\rangle$						2	-1			
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{1}\rangle$	2	2								
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{2}\rangle$			2		2					
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{3}\rangle$						2		2		2
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{4}\rangle$										-2
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{6}\rangle$										
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{1}\rangle$										
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{2}\rangle$										
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{3}\rangle$										
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{4}\rangle$										2
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{6}\rangle$										
	$\sqrt{72}$ each state									

Columns (100010), $\lvert 11\rangle$–$\lvert 20\rangle$:

	(100010)									
	$\lvert 11\rangle$	$\lvert 12\rangle$	$\lvert 13\rangle$	$\lvert 14\rangle$	$\lvert 15\rangle$	$\lvert 16\rangle$	$\lvert 17\rangle$	$\lvert 18\rangle$	$\lvert 19\rangle$	$\lvert 20\rangle$
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{1}\rangle$	-2				-2					
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{2}\rangle$	2				2	2	2		-2	-2
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{3}\rangle$	-2			2	-2					
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{4}\rangle$		1	1		2					
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{6}\rangle$		-1	1							
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{1}\rangle$					2					
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{2}\rangle$					-2		-2		2	
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{3}\rangle$				-2	2					
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{4}\rangle$		1			-1					
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{6}\rangle$		-1	-2		-1					
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{1}\rangle$	-2									
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{2}\rangle$	2					2			2	2
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{3}\rangle$	-2									
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{4}\rangle$		-2	-2	-2	-2					
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{6}\rangle$										
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{1}\rangle$	2				2					
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{2}\rangle$	-2				-2	-2	-2	-2	-2	
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{3}\rangle$	2			-2	2					
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{4}\rangle$	-1	1			-1					
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{6}\rangle$	-1	-1	-2		-1					
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{1}\rangle$										-2
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{2}\rangle$									-2	
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{3}\rangle$	2					-2			2	
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{4}\rangle$		2		-2	2		-2	-1	-1	
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{6}\rangle$								1	-1	
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{1}\rangle$	2									
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{2}\rangle$	-2					-2		2	2	
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{3}\rangle$	2									
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{4}\rangle$	-1	-2		2						
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{6}\rangle$	-1									
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{1}\rangle$										-2
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{2}\rangle$									-2	
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{3}\rangle$	-2					-2			2	
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{4}\rangle$		-2			-2		-2	-1	-1	
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{6}\rangle$								1	-1	
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{1}\rangle$										2
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{2}\rangle$								-2	-2	-2
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{3}\rangle$						2		2	2	2
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{4}\rangle$		2	2	2			2		-1	-1
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{6}\rangle$								-2	-1	-1
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{1}\rangle$										2
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{2}\rangle$								-2	-2	-2
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{3}\rangle$						2		2	2	2
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{4}\rangle$		-2	-2				2		-1	-1
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{6}\rangle$								-2	-1	-1
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{1}\rangle$									2	2
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{2}\rangle$								2		-2
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{3}\rangle$						-2		-2		2
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{4}\rangle$			2				-2			-1
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{6}\rangle$								2		-1
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{1}\rangle$									2	2
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{2}\rangle$								2		-2
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{3}\rangle$						-2		-2		2
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{4}\rangle$			-2	-2			-2			-1
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{6}\rangle$								2		-1
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{1}\rangle$	2									
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{2}\rangle$	-2									
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{3}\rangle$	2									
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{4}\rangle$	-1			-2						
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{6}\rangle$	-1					2		-2		
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{1}\rangle$	2	2			2					
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{2}\rangle$	-2		2		-2					
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{3}\rangle$	2			2	2					
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{4}\rangle$	-1				-1					
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{6}\rangle$	-1				-1	2	2	2		
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{1}\rangle$		2			2					
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{2}\rangle$			2		-2					
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{3}\rangle$				2	2					
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{4}\rangle$					-1					
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{6}\rangle$					-1		2			

Columns (000001):

	(000001)					
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{1}\rangle$			-2	-2	-2	-2
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{2}\rangle$	4					
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{3}\rangle$		4	2	2	2	2
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{4}\rangle$			1	-3	-1	-1
$\lvert 1\bar{1}01\bar{1}0\rangle\ \lvert\bar{1}10\bar{1}10_{6}\rangle$			-1	-1	1	-3
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{1}\rangle$				2		2
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{2}\rangle$	4					
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{3}\rangle$		4		-2		-2
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{4}\rangle$			4	3		1
$\lvert\bar{1}00010\rangle\ \lvert 1000\bar{1}0_{6}\rangle$				1	4	3
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{1}\rangle$		-2	-2		2	2
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{2}\rangle$	-4					
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{3}\rangle$		-2	2		-2	-2
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{4}\rangle$		1	1	4	1	1
$\lvert 1\bar{1}1\bar{1}00\rangle\ \lvert\bar{1}1\bar{1}100_{6}\rangle$		-1	-1		-1	3
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{1}\rangle$			2	2		-2
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{2}\rangle$	-4					
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{3}\rangle$		-4	-2	-2		2
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{4}\rangle$			-1	3		-1
$\lvert\bar{1}001\bar{1}0\rangle\ \lvert 100\bar{1}10_{6}\rangle$			1	1	4	-3
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{1}\rangle$	-2	-2			-4	-2
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{2}\rangle$	4					
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{3}\rangle$	-2	-2	-4			2
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{4}\rangle$	1	1		-4		-1
$\lvert 10\bar{1}001\rangle\ \lvert\bar{1}0100\bar{1}_{6}\rangle$	-1	-1				-3
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{1}\rangle$		2	2			2
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{2}\rangle$	4					
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{3}\rangle$		2	-2			-2
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{4}\rangle$		-1	-1	-4		1
$\lvert\bar{1}01\bar{1}00\rangle\ \lvert 10\bar{1}100_{6}\rangle$		1	1		4	3
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{1}\rangle$	-2				4	2
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{2}\rangle$	4	4				
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{3}\rangle$	-2		4			-2
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{4}\rangle$	1			4		1
$\lvert 10000\bar{1}\rangle\ \lvert\bar{1}00001_{6}\rangle$	-1					3
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{1}\rangle$	2	2			2	-2
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{2}\rangle$	-4					
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{3}\rangle$	2	2	4		2	2
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{4}\rangle$	-1	-1		4	-1	-1
$\lvert\bar{1}1\bar{1}001\rangle\ \lvert 1\bar{1}100\bar{1}_{6}\rangle$	1	1			-3	-3
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{1}\rangle$	2				-2	2
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{2}\rangle$	-4	-4				
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{3}\rangle$	2		-4		-2	-2
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{4}\rangle$	-1			-4	1	1
$\lvert\bar{1}1000\bar{1}\rangle\ \lvert 1\bar{1}0001_{6}\rangle$	1				3	3
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{1}\rangle$	-2				2	4
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{2}\rangle$	4					
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{3}\rangle$	-2		-4		2	
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{4}\rangle$	1			-4	-1	
$\lvert 0\bar{1}0001\rangle\ \lvert 01000\bar{1}_{6}\rangle$	-1	-4			-3	
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{1}\rangle$	-2	-2			-2	-4
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{2}\rangle$	4	4				
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{3}\rangle$	-2	-2	4		-2	
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{4}\rangle$	1	1		4	1	
$\lvert 0\bar{1}100\bar{1}\rangle\ \lvert 01\bar{1}001_{6}\rangle$	-1	3			3	
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{1}\rangle$		-2	-2			4
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{2}\rangle$		4	4		4	
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{3}\rangle$		-2	-6			
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{4}\rangle$		1	1	-4		
$\lvert 00\bar{1}100\rangle\ \lvert 001\bar{1}00_{6}\rangle$	4	3	3			
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{1}\rangle$			-2	-2		-4
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{2}\rangle$			4	4	-4	
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{3}\rangle$		-4	-6	-6		
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{4}\rangle$			1	5		
$\lvert 000\bar{1}10\rangle\ \lvert 0001\bar{1}0_{6}\rangle$	-4		3	3		
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{1}\rangle$				-2		4
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{2}\rangle$				4	4	
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{3}\rangle$		4		-6		
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{4}\rangle$			4	5		
$\lvert 0000\bar{1}0\rangle\ \lvert 000010_{6}\rangle$	4			3		
	$\sqrt{360}$ each state					

Table 24: **CG coefficients for (001000) dominant weight in (100000)⊗(200000).** Each entry should be divided by the respective number in the last row to keep the states normalized to 1.

	(300000)	*(110000)*	
	$\|001000\rangle$	$\|001000_1\rangle$	$\|001000_2\rangle$
$\|0\bar{1}1000\rangle$ $\|010000\rangle$	1	2	1
$\|\bar{1}10000\rangle$ $\|1\bar{1}1000\rangle$	1	-1	1
$\|100000\rangle$ $\|\bar{1}01000\rangle$	1	-1	-2
	$\sqrt{3}$	$\sqrt{6}$	$\sqrt{6}$

Table 25: **CG coefficients for (100010) dominant weight in (100000)⊗(200000).** $|\mathrm{n}\rangle$ is an abbreviation for $|100010_n\rangle$. Numbering of the degenerate states is consistent with tables 5 and 4. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(300000)				*(110000)*								*(100010)*
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|100010\rangle$
$\|100000\rangle$ $\|000010_1\rangle$				2	-$\sqrt{2}$								$\sqrt{32}$
$\|100000\rangle$ $\|000010_2\rangle$			$\sqrt{2}$			-$\sqrt{2}$	-$\sqrt{8}$						-$\sqrt{18}$
$\|100000\rangle$ $\|000010_3\rangle$		$\sqrt{2}$									-$\sqrt{2}$	-$\sqrt{8}$	$\sqrt{8}$
$\|100000\rangle$ $\|000010_6\rangle$	$\sqrt{2}$							-$\sqrt{8}$	-$\sqrt{2}$	-$\sqrt{2}$			-$\sqrt{2}$
$\|\bar{1}10000\rangle$ $\|2\bar{1}0010\rangle$			1	$\sqrt{2}$	-1	-1	1						-5
$\|0\bar{1}1000\rangle$ $\|11\bar{1}010\rangle$		1	1			-1	1				-1	1	5
$\|00\bar{1}101\rangle$ $\|101\bar{1}1\bar{1}\rangle$	1	1						1	-1	-1	-1	1	-5
$\|000\bar{1}11\rangle$ $\|10010\bar{1}\rangle$	1							1	2	-1			5
$\|00010\bar{1}\rangle$ $\|100\bar{1}11\rangle$	1							1	-1	2			5
$\|001\bar{1}1\bar{1}\rangle$ $\|10\bar{1}101\rangle$	1	1						1	2	2	2	1	-5
$\|01\bar{1}010\rangle$ $\|1\bar{1}1000\rangle$		1	1			2	1				2	1	5
$\|1\bar{1}0010\rangle$ $\|010000\rangle$			1	$\sqrt{2}$	2	2	1						-5
$\|\bar{1}00010\rangle$ $\|200000\rangle$				1	$\sqrt{2}$								$\sqrt{50}$
	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	3	3	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{270}$

Table 26: **CG coefficients for (000001) dominant weight states of the 3003-dimensional (300000) irrep and 650-dimensional (100010) irrep in (100000)⊗(200000).** $|n\rangle$ is an abbreviation for $|000001_n\rangle$. Numbering of the degenerate states is consistent with table 5, and table I in [5]. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(300000)										*(100010)*				
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$
$\vert 100000\rangle\,\vert\bar{1}00001_1\rangle$										$\sqrt{2}$	$\sqrt{32}$	$\sqrt{50}$			
$\vert 100000\rangle\,\vert\bar{1}00001_2\rangle$								$\sqrt{2}$			$-\sqrt{18}$		$\sqrt{50}$		
$\vert 100000\rangle\,\vert\bar{1}00001_3\rangle$						$\sqrt{2}$					$\sqrt{8}$			$\sqrt{50}$	
$\vert 100000\rangle\,\vert\bar{1}00001_6\rangle$				$\sqrt{2}$							$-\sqrt{2}$				$\sqrt{50}$
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_1\rangle$										$\sqrt{2}$	$\sqrt{32}$	$-\sqrt{2}$			
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_2\rangle$								$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{18}$	$-\sqrt{18}$	$-\sqrt{50}$		
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_3\rangle$					$\sqrt{2}$	$\sqrt{2}$					$\sqrt{8}$	$\sqrt{8}$		$-\sqrt{50}$	
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_6\rangle$			$\sqrt{2}$	$\sqrt{2}$							$-\sqrt{2}$	$-\sqrt{2}$			$-\sqrt{50}$
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_1\rangle$								$\sqrt{2}$		$\sqrt{2}$	$-\sqrt{50}$	$-\sqrt{2}$	$-\sqrt{2}$		
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_2\rangle$									$\sqrt{2}$	$\sqrt{2}$		$-\sqrt{18}$	$\sqrt{8}$		
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_3\rangle$					$\sqrt{2}$		$\sqrt{2}$		$\sqrt{2}$			$\sqrt{8}$	$\sqrt{8}$	$\sqrt{50}$	
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_6\rangle$		$\sqrt{2}$	$\sqrt{2}$									$-\sqrt{2}$	$-\sqrt{2}$		$\sqrt{50}$
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_1\rangle$						$\sqrt{2}$		$\sqrt{2}$			$\sqrt{50}$		$-\sqrt{2}$	$-\sqrt{2}$	
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_2\rangle$					$\sqrt{2}$				$\sqrt{2}$	$\sqrt{2}$		$\sqrt{50}$	$\sqrt{8}$	$\sqrt{8}$	
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_3\rangle$							$\sqrt{2}$		$\sqrt{2}$				$\sqrt{8}$	$-\sqrt{18}$	
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_6\rangle$	$\sqrt{2}$	$\sqrt{2}$					$\sqrt{2}$						$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{50}$
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_1\rangle$				$\sqrt{2}$		$\sqrt{2}$					$-\sqrt{50}$			$-\sqrt{2}$	$-\sqrt{2}$
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_2\rangle$			$\sqrt{2}$		$\sqrt{2}$							$-\sqrt{50}$		$\sqrt{8}$	$\sqrt{8}$
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_3\rangle$		$\sqrt{2}$					$\sqrt{2}$		$\sqrt{2}$				$-\sqrt{50}$	$-\sqrt{18}$	$-\sqrt{18}$
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_6\rangle$	$\sqrt{2}$						$\sqrt{2}$							$-\sqrt{2}$	$\sqrt{32}$
$\vert 0000\bar{1}1\rangle\,\vert 000010_1\rangle$				$\sqrt{2}$							$\sqrt{50}$				$-\sqrt{2}$
$\vert 0000\bar{1}1\rangle\,\vert 000010_2\rangle$			$\sqrt{2}$									$\sqrt{50}$			$\sqrt{8}$
$\vert 0000\bar{1}1\rangle\,\vert 000010_3\rangle$		$\sqrt{2}$											$\sqrt{50}$		$-\sqrt{18}$
$\vert 0000\bar{1}1\rangle\,\vert 000010_6\rangle$	$\sqrt{2}$						$\sqrt{2}$							$\sqrt{50}$	$\sqrt{32}$
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}012\rangle$	1	1					2		1				-5	-10	-5
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}2\rangle$	1	1					1						5	5	-5
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}02\rangle$	1						1							5	10
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}1\rangle$		1	1									5	5		5
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}1\rangle$			1	1							5	5			-5
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}11\rangle$		1	1		1		1		1			-5	-5	5	5
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0101\rangle$					1		1		2	1		5	10	5	
$\vert 1\bar{1}01\bar{1}0\rangle\,\vert\bar{1}10\bar{1}11\rangle$			1	1	1	1					-5	-5		-5	-5
$\vert\bar{1}00010\rangle\,\vert 1000\bar{1}1\rangle$				1							5				5
$\vert 1\bar{1}1\bar{1}00\rangle\,\vert\bar{1}1\bar{1}101\rangle$					1	1		1	1	1	5	5	-5	-5	
$\vert\bar{1}001\bar{1}0\rangle\,\vert 100\bar{1}11\rangle$				1		1					-5			5	5
$\vert 10\bar{1}001\rangle\,\vert\bar{1}01000\rangle$								1	1	2	-5	-10	-5		
$\vert\bar{1}01\bar{1}00\rangle\,\vert 10\bar{1}101\rangle$						1		1			5		5	5	
$\vert\bar{1}1\bar{1}001\rangle\,\vert 1\bar{1}1000\rangle$								1		1	-5	5	5		
$\vert 0\bar{1}0001\rangle\,\vert 010000\rangle$										1	10	5			
	3	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{24}$	$\sqrt{12}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{540}$	$\sqrt{540}$	$\sqrt{540}$	$\sqrt{540}$	$\sqrt{540}$

Table 27: **CG coefficients for (000001) dominant weight states of the 5824-dimensional (110000) irrep in (100000)⊗(200000).** |n⟩ is an abbreviation for $|000001_n\rangle$. Numbering of the degenerate states is consistent with table 4. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(110000)																							
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$	$\vert 21\rangle$	$\vert 22\rangle$	$\vert 23\rangle$	$\vert 24\rangle$
$\vert 100000\rangle\,\vert\bar{1}00001_1\rangle$																					-1	$-\sqrt{8}$		
$\vert 100000\rangle\,\vert\bar{1}00001_2\rangle$																	-1	$-\sqrt{8}$						
$\vert 100000\rangle\,\vert\bar{1}00001_3\rangle$										-1	$-\sqrt{8}$													
$\vert 100000\rangle\,\vert\bar{1}00001_6\rangle$							$-\sqrt{8}$	-1																$-\sqrt{2}$
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_1\rangle$																					-1	$\sqrt{2}$		
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_2\rangle$																	-1	$-\sqrt{8}$	-1	$-\sqrt{8}$		$-\sqrt{8}$		
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_3\rangle$										-1	$-\sqrt{8}$	-1	$-\sqrt{8}$											
$\vert\bar{1}10000\rangle\,\vert 1\bar{1}0001_6\rangle$					$-\sqrt{8}$	-1	$-\sqrt{8}$	-1															$-\sqrt{2}$	$-\sqrt{2}$
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_1\rangle$																	-1	$\sqrt{2}$			-1	$\sqrt{2}$		
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_2\rangle$																			-1	$\sqrt{2}$		$-\sqrt{8}$		
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_3\rangle$												-1	$-\sqrt{8}$	-1	$-\sqrt{8}$					$-\sqrt{8}$				
$\vert 0\bar{1}1000\rangle\,\vert 01\bar{1}001_6\rangle$			$-\sqrt{8}$	-1	$-\sqrt{8}$	-1										$-\sqrt{2}$							$-\sqrt{2}$	
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_1\rangle$										-1	$\sqrt{2}$						-1	$\sqrt{2}$						
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_2\rangle$												-1	$\sqrt{2}$						-1	$\sqrt{2}$		$\sqrt{2}$		
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_3\rangle$														-1	$\sqrt{2}$					$-\sqrt{8}$				
$\vert 00\bar{1}101\rangle\,\vert 001\bar{1}00_6\rangle$	$-\sqrt{8}$	$-\sqrt{2}$	$-\sqrt{8}$	-1					-1						$-\sqrt{8}$	$-\sqrt{2}$								
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_1\rangle$							$\sqrt{2}$	-1		-1	$\sqrt{2}$													$-\sqrt{2}$
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_2\rangle$					$\sqrt{2}$	-1						-1	$\sqrt{2}$										$-\sqrt{2}$	
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_3\rangle$			$\sqrt{2}$	-1										-1	$\sqrt{2}$	$-\sqrt{2}$				$\sqrt{2}$				
$\vert 000\bar{1}11\rangle\,\vert 0001\bar{1}0_6\rangle$	$\sqrt{2}$	$-\sqrt{2}$							-1						$-\sqrt{8}$									
$\vert 0000\bar{1}1\rangle\,\vert 000010_1\rangle$							$\sqrt{2}$	-1																$\sqrt{8}$
$\vert 0000\bar{1}1\rangle\,\vert 000010_2\rangle$					$\sqrt{2}$	-1																	$\sqrt{8}$	
$\vert 0000\bar{1}1\rangle\,\vert 000010_3\rangle$			$\sqrt{2}$	-1												$\sqrt{8}$								
$\vert 0000\bar{1}1\rangle\,\vert 000010_6\rangle$	$\sqrt{2}$	$\sqrt{8}$							-1						$\sqrt{2}$									
$\vert 0010\bar{1}\bar{1}\rangle\,\vert 00\bar{1}012\rangle$	1	2	1	$\sqrt{2}$					$\sqrt{2}$					$\sqrt{2}$	2	2				1				
$\vert 001\bar{1}1\bar{1}\rangle\,\vert 00\bar{1}1\bar{1}2\rangle$	1	-1	1	$\sqrt{2}$					$\sqrt{2}$						1	-1								
$\vert 00010\bar{1}\rangle\,\vert 000\bar{1}02\rangle$	-2	-1							$\sqrt{2}$						1									
$\vert 01\bar{1}010\rangle\,\vert 0\bar{1}10\bar{1}1\rangle$			1	$\sqrt{2}$	1	$\sqrt{2}$										-1							-1	
$\vert 1\bar{1}0010\rangle\,\vert\bar{1}100\bar{1}1\rangle$					1	$\sqrt{2}$	1	$\sqrt{2}$															-1	-1
$\vert 01\bar{1}1\bar{1}0\rangle\,\vert 0\bar{1}1\bar{1}11\rangle$			1	$\sqrt{2}$	1	$\sqrt{2}$						$\sqrt{2}$	1	$\sqrt{2}$	1	2				1			2	
$\vert 010\bar{1}00\rangle\,\vert 0\bar{1}0101\rangle$												$\sqrt{2}$	1	$\sqrt{2}$	1				$\sqrt{2}$	2		1		
$\vert 1\bar{1}01\bar{1}0\rangle\,\vert\bar{1}10\bar{1}11\rangle$					1	$\sqrt{2}$	1	$\sqrt{2}$		$\sqrt{2}$	1	$\sqrt{2}$	1										2	2
$\vert\bar{1}00010\rangle\,\vert 1000\bar{1}1\rangle$							1	$\sqrt{2}$																-1
$\vert 1\bar{1}1\bar{1}00\rangle\,\vert\bar{1}1\bar{1}101\rangle$										$\sqrt{2}$	1	$\sqrt{2}$	1				$\sqrt{2}$	1	$\sqrt{2}$	1		1		
$\vert\bar{1}001\bar{1}0\rangle\,\vert 100\bar{1}11\rangle$							1	$\sqrt{2}$		$\sqrt{2}$	1													2
$\vert 10\bar{1}001\rangle\,\vert\bar{1}01000\rangle$																	$\sqrt{2}$	1	$\sqrt{2}$	1	$\sqrt{2}$	2		
$\vert\bar{1}01\bar{1}00\rangle\,\vert 10\bar{1}101\rangle$										$\sqrt{2}$	1						$\sqrt{2}$	1						
$\vert\bar{1}1\bar{1}001\rangle\,\vert 1\bar{1}1000\rangle$																	$\sqrt{2}$	1			$\sqrt{2}$	1		
$\vert 0\bar{1}0001\rangle\,\vert 010000\rangle$																					$\sqrt{2}$	1		
	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{24}$	$\sqrt{12}$	$\sqrt{24}$	$\sqrt{12}$	$\sqrt{24}$	$\sqrt{12}$	3	$\sqrt{12}$	$\sqrt{24}$	$\sqrt{12}$	$\sqrt{24}$	3	$\sqrt{30}$	$\sqrt{24}$	$\sqrt{12}$	$\sqrt{24}$	3	$\sqrt{30}$	3	$\sqrt{30}$	$\sqrt{24}$	$\sqrt{24}$

Table 28: **CG coefficients for the (000000) dominant weight states of the 3003-dimensional (300000) irrep in the product (100000)⊗(200000).** $\lvert n\rangle$ is an abbreviation for $\lvert 000000_n\rangle$. Numbering of the degenerate states is consistent with table 5. Each CGC should be divided by the respective number in the last row of the table to maintain $\langle n\vert n\rangle = 1$.

	(300000)																							
	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$	$\lvert 7\rangle$	$\lvert 8\rangle$	$\lvert 9\rangle$	$\lvert 10\rangle$	$\lvert 11\rangle$	$\lvert 12\rangle$	$\lvert 13\rangle$	$\lvert 14\rangle$	$\lvert 15\rangle$	$\lvert 16\rangle$	$\lvert 17\rangle$	$\lvert 18\rangle$	$\lvert 19\rangle$	$\lvert 20\rangle$	$\lvert 21\rangle$	$\lvert 22\rangle$	$\lvert 23\rangle$	$\lvert 24\rangle$
$\lvert 100000\rangle\ \lvert\bar{1}00000_1\rangle$															1									
$\lvert 100000\rangle\ \lvert\bar{1}00000_2\rangle$																					1			
$\lvert 100000\rangle\ \lvert\bar{1}00000_3\rangle$																							1	
$\lvert 100000\rangle\ \lvert\bar{1}00000_6\rangle$						1													1					
$\lvert\bar{1}10000\rangle\ \lvert 1\bar{1}0000_1\rangle$												1			1									
$\lvert\bar{1}10000\rangle\ \lvert 1\bar{1}0000_2\rangle$																	1				1			
$\lvert\bar{1}10000\rangle\ \lvert 1\bar{1}0000_3\rangle$																			1				1	
$\lvert\bar{1}10000\rangle\ \lvert 1\bar{1}0000_6\rangle$						1	1												1					
$\lvert 0\bar{1}1000\rangle\ \lvert 01\bar{1}000_1\rangle$												1				1								
$\lvert 0\bar{1}1000\rangle\ \lvert 01\bar{1}000_2\rangle$																	1					1		
$\lvert 0\bar{1}1000\rangle\ \lvert 01\bar{1}000_3\rangle$																			1					1
$\lvert 0\bar{1}1000\rangle\ \lvert 01\bar{1}000_6\rangle$						1	1			1									1				1	
$\lvert 00\bar{1}101\rangle\ \lvert 001\bar{1}0\bar{1}_1\rangle$				1												1								
$\lvert 00\bar{1}101\rangle\ \lvert 001\bar{1}0\bar{1}_2\rangle$									1													1		
$\lvert 00\bar{1}101\rangle\ \lvert 001\bar{1}0\bar{1}_3\rangle$					1		1										1		1					
$\lvert 00\bar{1}101\rangle\ \lvert 001\bar{1}0\bar{1}_6\rangle$								1		1											1		1	1
$\lvert 000\bar{1}11\rangle\ \lvert 0001\bar{1}\bar{1}_1\rangle$				1																				
$\lvert 000\bar{1}11\rangle\ \lvert 0001\bar{1}\bar{1}_2\rangle$			1	5					1							11						1		1
$\lvert 000\bar{1}11\rangle\ \lvert 0001\bar{1}\bar{1}_3\rangle$	1				1							1					1							
$\lvert 000\bar{1}11\rangle\ \lvert 0001\bar{1}\bar{1}_6\rangle$		1						1							1						1			
$\lvert 00010\bar{1}\rangle\ \lvert 000\bar{1}01_1\rangle$				1																				
$\lvert 00010\bar{1}\rangle\ \lvert 000\bar{1}01_2\rangle$									1															
$\lvert 00010\bar{1}\rangle\ \lvert 000\bar{1}01_3\rangle$					1		1											1						
$\lvert 00010\bar{1}\rangle\ \lvert 000\bar{1}01_6\rangle$								1		1										1				1
$\lvert 0000\bar{1}1\rangle\ \lvert 00001\bar{1}_1\rangle$				1					1															
$\lvert 0000\bar{1}1\rangle\ \lvert 00001\bar{1}_2\rangle$			1	5												11								
$\lvert 0000\bar{1}1\rangle\ \lvert 00001\bar{1}_3\rangle$	1											1												
$\lvert 0000\bar{1}1\rangle\ \lvert 00001\bar{1}_6\rangle$		1													1									
$\lvert 001\bar{1}1\bar{1}\rangle\ \lvert 00\bar{1}1\bar{1}1_1\rangle$				1												1								
$\lvert 001\bar{1}1\bar{1}\rangle\ \lvert 00\bar{1}1\bar{1}1_2\rangle$			1	5					1							10								
$\lvert 001\bar{1}1\bar{1}\rangle\ \lvert 00\bar{1}1\bar{1}1_3\rangle$	1				1								1					1						1
$\lvert 001\bar{1}1\bar{1}\rangle\ \lvert 00\bar{1}1\bar{1}1_6\rangle$		1						1						1						1				
$\lvert 0010\bar{1}\bar{1}\rangle\ \lvert 00\bar{1}011_1\rangle$				1					1							1						1		1
$\lvert 0010\bar{1}\bar{1}\rangle\ \lvert 00\bar{1}011_2\rangle$			1	5												10								
$\lvert 0010\bar{1}\bar{1}\rangle\ \lvert 00\bar{1}011_3\rangle$	1												1											
$\lvert 0010\bar{1}\bar{1}\rangle\ \lvert 00\bar{1}011_6\rangle$		1												1										
$\lvert 01\bar{1}010\rangle\ \lvert 0\bar{1}10\bar{1}0_1\rangle$												1				1								
$\lvert 01\bar{1}010\rangle\ \lvert 0\bar{1}10\bar{1}0_2\rangle$	1		1	5												10								
$\lvert 01\bar{1}010\rangle\ \lvert 0\bar{1}10\bar{1}0_3\rangle$													1											1
$\lvert 01\bar{1}010\rangle\ \lvert 0\bar{1}10\bar{1}0_6\rangle$											1		1	1					3	2				
$\lvert 01\bar{1}1\bar{1}0\rangle\ \lvert 0\bar{1}1\bar{1}10_1\rangle$												1				1	1					1		
$\lvert 01\bar{1}1\bar{1}0\rangle\ \lvert 0\bar{1}1\bar{1}10_2\rangle$	1		1	5	1				1							10								1
$\lvert 01\bar{1}1\bar{1}0\rangle\ \lvert 0\bar{1}1\bar{1}10_3\rangle$													1					1						
$\lvert 01\bar{1}1\bar{1}0\rangle\ \lvert 0\bar{1}1\bar{1}10_6\rangle$											1		1	1				2		1				
$\lvert 1\bar{1}0010\rangle\ \lvert\bar{1}100\bar{1}0_1\rangle$												1			1									
$\lvert 1\bar{1}0010\rangle\ \lvert\bar{1}100\bar{1}0_2\rangle$	1	1																						
$\lvert 1\bar{1}0010\rangle\ \lvert\bar{1}100\bar{1}0_3\rangle$													1	1										
$\lvert 1\bar{1}0010\rangle\ \lvert\bar{1}100\bar{1}0_6\rangle$											1		1					3		1				
$\lvert 010\bar{1}00\rangle\ \lvert 0\bar{1}0100_1\rangle$																	1		1			1		1
$\lvert 010\bar{1}00\rangle\ \lvert 0\bar{1}0100_2\rangle$					1		1		1															
$\lvert 010\bar{1}00\rangle\ \lvert 0\bar{1}0100_3\rangle$																		1						
$\lvert 010\bar{1}00\rangle\ \lvert 0\bar{1}0100_6\rangle$											1		1					2		1				

continued on next page

continued from previous page

	(300000)																							
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|16\rangle$	$\|17\rangle$	$\|18\rangle$	$\|19\rangle$	$\|20\rangle$	$\|21\rangle$	$\|22\rangle$	$\|23\rangle$	$\|24\rangle$
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_1\rangle$												1			1		1				1			
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_2\rangle$	1	1			1			1																
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_3\rangle$													1	1				1		1				1
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_6\rangle$											1		1					2		1				
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_1\rangle$															1									
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_2\rangle$		1																						
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_3\rangle$														1										
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_6\rangle$											1		1					2		1				
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_1\rangle$																	1		1		1		1	
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_2\rangle$					1		1	1		1														1
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_3\rangle$																		1		1				
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_6\rangle$											1		1	1				2		1				
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_1\rangle$															1						1			
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_2\rangle$		1						1																
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_3\rangle$														1						1				
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_6\rangle$											1		1					3		1				
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_1\rangle$						1	1												2				1	
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_2\rangle$										1														1
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_3\rangle$								1												1				
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_6\rangle$		1												1										
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_1\rangle$																					1		1	
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_2\rangle$								1		1														
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_3\rangle$																				1				1
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_6\rangle$											1		2	1				3		1				
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_1\rangle$						1	1												1					
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_2\rangle$										1													1	
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_3\rangle$								1													1			
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_6\rangle$		1													1									
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_1\rangle$						1													1				1	
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_2\rangle$							1			1														
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_3\rangle$					1			1										1		1				1
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_6\rangle$	1	1											1	1										
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_1\rangle$						1													1					
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_2\rangle$							1			1									1				1	1
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_3\rangle$					1			1									1				1			
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_6\rangle$	1	1										1			1									
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_1\rangle$						1													1					
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_2\rangle$							1																	
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_3\rangle$					1				1									1						
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_6\rangle$	1		1	5									1			10								1
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_1\rangle$						1													1				1	
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_2\rangle$							1												1					
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_3\rangle$					1				1								1					1		1
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_6\rangle$	1		1	5								1				11								
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_1\rangle$																					1		1	
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_2\rangle$																	1		1					
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_3\rangle$																						1		1
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_6\rangle$			1	6					1							11								
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_1\rangle$															1						1			
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_2\rangle$												1					1							
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_3\rangle$																1						1		
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_6\rangle$				1					1															
$\|0000\bar{1}0\rangle$ $\|000010_1\rangle$															1									
$\|0000\bar{1}0\rangle$ $\|000010_2\rangle$												1												
$\|0000\bar{1}0\rangle$ $\|000010_3\rangle$																1								
$\|0000\bar{1}0\rangle$ $\|000010_6\rangle$				1																				
	$\sqrt{12}$	$\sqrt{12}$	3	$\sqrt{264}$	$\sqrt{12}$	3	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{12}$	3	3	$\sqrt{12}$	$\sqrt{24}$	$\sqrt{12}$	$\sqrt{12}$	$\sqrt{1032}$	$\sqrt{12}$	$\sqrt{72}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{12}$	3	$\sqrt{12}$	$\sqrt{18}$

Table 29: **CG coefficients for the first 32 (000000) dominant weight states of the 5824-dimensional (110000) irrep in the product (100000)⊗(200000).** (The remaining 32 states of this irrep with the same weight are shown in table 30.) $|n\rangle$ is an abbreviation for $|000000_n\rangle$. Numbering of the degenerate states is consistent with table 4. Each CGC should be divided by the respective number in the last row of the table to maintain $\langle n|n\rangle = 1$.

	(110000)																															
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$	$\|7\rangle$	$\|8\rangle$	$\|9\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|14\rangle$	$\|15\rangle$	$\|16\rangle$	$\|17\rangle$	$\|18\rangle$	$\|19\rangle$	$\|20\rangle$	$\|21\rangle$	$\|22\rangle$	$\|23\rangle$	$\|24\rangle$	$\|25\rangle$	$\|26\rangle$	$\|27\rangle$	$\|28\rangle$	$\|29\rangle$	$\|30\rangle$	$\|31\rangle$	$\|32\rangle$
$\|100000\rangle\ \|\bar{1}00000_1\rangle$																																
$\|100000\rangle\ \|\bar{1}00000_2\rangle$																																
$\|100000\rangle\ \|\bar{1}00000_3\rangle$																																
$\|100000\rangle\ \|\bar{1}00000_6\rangle$											-2			-2								-1										
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_1\rangle$																													-1	-2		
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_2\rangle$																																
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_3\rangle$																																
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_6\rangle$											-2		-2	-3								-1										
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_1\rangle$																													-1	-2		
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_2\rangle$																																
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_3\rangle$																																
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_6\rangle$											-2		-2	-3							-2	-2		-1								
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_1\rangle$							-1	-2																								
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_2\rangle$																	-1	-2														
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_3\rangle$									-1	-2			-2	-1																		
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_6\rangle$															-1	-2			-1		-2	-1		-1								
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_1\rangle$							-1	1																								
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_2\rangle$					-1		-5	-4									-1	-2					-2									
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_3\rangle$	-1	-2							-1	-2		-2																	-11	-7		
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_6\rangle$			-1	-2		-1									-1	-2			-1	-2												
$\|00010\bar{1}\rangle\ \|000\bar{1}01_1\rangle$							-1	-2																								
$\|00010\bar{1}\rangle\ \|000\bar{1}01_2\rangle$																	-1	-2														
$\|00010\bar{1}\rangle\ \|000\bar{1}01_3\rangle$									-1	-2			-2	-1																		
$\|00010\bar{1}\rangle\ \|000\bar{1}01_6\rangle$															-1	-2			-1		-2	-1		-1								
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_1\rangle$							-1	1									-1	1														
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_2\rangle$					-1		-5	-4															1									
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_3\rangle$	-1	-2										1																	-11	-7		
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_6\rangle$			-1	-2		-1														1												
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_1\rangle$							-1	1																								
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_2\rangle$					-1		-5	-4									-1	-2					-2									
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_3\rangle$	-1	-2							-1	-2		-2													-1	-1	-2	-1	-10	-5		
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_6\rangle$			-1	-2		-1									-1	-2			-1	-2											-1	-2
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_1\rangle$							-1	1									-1	1														
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_2\rangle$					-1		-5	-4															1									
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_3\rangle$	-1	-2										1													-1	-1	-2	-1	-10	-5		
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_6\rangle$			-1	-2		-1														1											-1	-2
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_1\rangle$																													-1	1		
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_2\rangle$	-1	1			-1		-5	-1				-2											-2						-10	-5		
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_3\rangle$																									-1	-1	1	-1				
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_6\rangle$																										-1	-2	-1			-1	-2
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_1\rangle$																													-1	1		
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_2\rangle$	-1	1			-1		-5	-1	-1	1		1					-1	1					1						-10	-5		
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_3\rangle$																									-1	-1	1	-1				
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_6\rangle$																										-1	-2	-1			-1	-2
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_1\rangle$																													-1	1		
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_2\rangle$	-1	1	-1	1		-1						-2								-2									-10	-5		
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_3\rangle$																									-1	-1	1	-1			-1	1
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_6\rangle$																										-1	-2	-1				
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_1\rangle$																																
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_2\rangle$									-1	1			1	-1			-1	1														
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_3\rangle$																																
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_6\rangle$																										2	1	-1				

continued on next page

continued from previous page

	(110000)															
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_1\rangle$																
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_2\rangle$	-1	1	-1	1		-1			-1	1		1			-1	1
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_3\rangle$																
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_6\rangle$																
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_1\rangle$																
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_2\rangle$			-1	1		2										
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_3\rangle$																
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_6\rangle$																
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_1\rangle$																
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_2\rangle$									-1	1			1	-1	-1	1
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_3\rangle$																
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_6\rangle$																
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_1\rangle$																
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_2\rangle$			-1	1		2									1	-1
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_3\rangle$																
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_6\rangle$																
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_1\rangle$											1		1			
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_2\rangle$																
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_3\rangle$															2	1
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_6\rangle$			2	1		-1										
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_1\rangle$																
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_2\rangle$															-1	1
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_3\rangle$																
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_6\rangle$																
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_1\rangle$											1		1			
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_2\rangle$																
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_3\rangle$															2	1
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_6\rangle$			2	1		-1										
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_1\rangle$											1			1		
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_2\rangle$													1	2		
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_3\rangle$									2	1					2	1
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_6\rangle$	2	1	2	1		2						1				
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_1\rangle$											1			1		
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_2\rangle$													1	2		
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_3\rangle$									2	1					2	1
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_6\rangle$	2	1	2	1		2						1				
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_1\rangle$											1			1		
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_2\rangle$													1	2		
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_3\rangle$									2	1						
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_6\rangle$	2	1			2		10	5				1				
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_1\rangle$											1			1		
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_2\rangle$													1	2		
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_3\rangle$									2	1						
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_6\rangle$	2	1		2		10	5				1					
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_1\rangle$																
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_2\rangle$																
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_3\rangle$																
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_6\rangle$					2		12	6								
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_1\rangle$																
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_2\rangle$																
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_3\rangle$																
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_6\rangle$							2	1								
$\vert 0000\bar{1}0\rangle\ \vert 000010_1\rangle$																
$\vert 0000\bar{1}0\rangle\ \vert 000010_2\rangle$																
$\vert 0000\bar{1}0\rangle\ \vert 000010_3\rangle$																
$\vert 0000\bar{1}0\rangle\ \vert 000010_6\rangle$							2	1								
	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{24}$	$\sqrt{528}$	$\sqrt{150}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{54}$	$\sqrt{24}$	$\sqrt{24}$

	(110000)															
	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$	$\vert 21\rangle$	$\vert 22\rangle$	$\vert 23\rangle$	$\vert 24\rangle$	$\vert 25\rangle$	$\vert 26\rangle$	$\vert 27\rangle$	$\vert 28\rangle$	$\vert 29\rangle$	$\vert 30\rangle$	$\vert 31\rangle$	$\vert 32\rangle$
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_1\rangle$													-1	1		
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_2\rangle$			-1	1									-10	-5		
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_3\rangle$									-1	-1	1	-1			-1	1
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_6\rangle$										-1	-2	-1				
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_1\rangle$																
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_2\rangle$				-2												
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_3\rangle$															-1	1
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_6\rangle$										-1	1	2				
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_1\rangle$																
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_2\rangle$			-1		1	-1		-1								
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_3\rangle$																
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_6\rangle$										2	1	-1			2	1
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_1\rangle$																
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_2\rangle$			2	1												
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_3\rangle$															-1	1
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_6\rangle$										-1	1	2				
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_1\rangle$						-1										
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_2\rangle$					1	2		-1								
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_3\rangle$			-1													
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_6\rangle$				1											2	1
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_1\rangle$																
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_2\rangle$			2		1	-1		2								
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_3\rangle$																
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_6\rangle$									2	4	2	4			2	1
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_1\rangle$						-1										
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_2\rangle$					1	2		-1								
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_3\rangle$			-1													
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_6\rangle$				1												
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_1\rangle$						-1										
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_2\rangle$					1	2		2								
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_3\rangle$			2													
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_6\rangle$				1					2	2	1	2	20	10	2	1
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_1\rangle$						-1										
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_2\rangle$					1	2		2								
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_3\rangle$			2													
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_6\rangle$				1									22	11		
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_1\rangle$						2										
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_2\rangle$																
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_3\rangle$	2	1														
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_6\rangle$							1		2	2	1	2	20	10		
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_1\rangle$						2										
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_2\rangle$																
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_3\rangle$	2	1														
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_6\rangle$						1						22 11				
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_1\rangle$																
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_2\rangle$																
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_3\rangle$																
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_6\rangle$	2	1					1									
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_1\rangle$																
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_2\rangle$													2	1		
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_3\rangle$																
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_6\rangle$	2	1														
$\vert 0000\bar{1}0\rangle\ \vert 000010_1\rangle$																
$\vert 0000\bar{1}0\rangle\ \vert 000010_2\rangle$													2	1		
$\vert 0000\bar{1}0\rangle\ \vert 000010_3\rangle$																
$\vert 0000\bar{1}0\rangle\ \vert 000010_6\rangle$																
	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{30}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{48}$	$\sqrt{30}$	$\sqrt{48}$	$\sqrt{2664}$	$\sqrt{684}$	$\sqrt{24}$	$\sqrt{24}$

Table 30: **CG coefficients for the remaining 32 (000000) dominant weight states of the 5824-dimensional (110000) irrep in the product (100000)⊗(200000).** (The first 32 states of this irrep with the same weight are shown in table 29.) |n⟩ is an abbreviation for $|000000_n\rangle$. Numbering of the degenerate states is consistent with table 4. Each CGC should be divided by the respective number in the last row to maintain $\langle n|n\rangle = 1$.

	(110000)																															
	$\|33\rangle$	$\|34\rangle$	$\|35\rangle$	$\|36\rangle$	$\|37\rangle$	$\|38\rangle$	$\|39\rangle$	$\|40\rangle$	$\|41\rangle$	$\|42\rangle$	$\|43\rangle$	$\|44\rangle$	$\|45\rangle$	$\|46\rangle$	$\|47\rangle$	$\|48\rangle$	$\|49\rangle$	$\|50\rangle$	$\|51\rangle$	$\|52\rangle$	$\|53\rangle$	$\|54\rangle$	$\|55\rangle$	$\|56\rangle$	$\|57\rangle$	$\|58\rangle$	$\|59\rangle$	$\|60\rangle$	$\|61\rangle$	$\|62\rangle$	$\|63\rangle$	$\|64\rangle$
$\|100000\rangle\ \|\bar{1}00000_1\rangle$		-2	-1																													
$\|100000\rangle\ \|\bar{1}00000_2\rangle$																					-2	-1										
$\|100000\rangle\ \|\bar{1}00000_3\rangle$																												-2	-1			
$\|100000\rangle\ \|\bar{1}00000_6\rangle$																	-1															
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_1\rangle$		-2	-1																													
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_2\rangle$												-1	-2								-2	-1										
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_3\rangle$															-1	-2												-2	-1			-2
$\|\bar{1}10000\rangle\ \|1\bar{1}0000_6\rangle$																	-1	-1														
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_1\rangle$						-1	-2																									
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_2\rangle$												-1	-2										-1	-2								
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_3\rangle$															-1	-2															-2	-3
$\|0\bar{1}1000\rangle\ \|01\bar{1}000_6\rangle$																	-1	-1									-1	-2	-1			-2
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_1\rangle$						-1	-2	-2																								
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_2\rangle$																							-1	-2		-2						
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_3\rangle$												-1	-2		-1	-2		-1													-2	-3
$\|00\bar{1}101\rangle\ \|001\bar{1}0\bar{1}_6\rangle$																					-2	-1					-1	-2	-1			-2
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_1\rangle$								-2																								
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_2\rangle$				-2		-11	-7																-1	-2		-2					-2	-1
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_3\rangle$												-1	-2																			
$\|000\bar{1}11\rangle\ \|0001\bar{1}\bar{1}_6\rangle$		-2	-1																		-2	-1										
$\|00010\bar{1}\rangle\ \|000\bar{1}01_1\rangle$								1																								
$\|00010\bar{1}\rangle\ \|000\bar{1}01_2\rangle$																										1						
$\|00010\bar{1}\rangle\ \|000\bar{1}01_3\rangle$										-2	-1							-2								-2						
$\|00010\bar{1}\rangle\ \|000\bar{1}01_6\rangle$																			-2	-1					-1		-2			-4	-2	-3
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_1\rangle$								-2																		-2						
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_2\rangle$				-2		-11	-7																									
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_3\rangle$																																
$\|0000\bar{1}1\rangle\ \|00001\bar{1}_6\rangle$		-2	-1																													
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_1\rangle$						-1	1	1																								
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_2\rangle$				1		-10	-5																			1						
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_3\rangle$				-2						-2	-1			-2				-1								-2					-2	-1
$\|001\bar{1}1\bar{1}\rangle\ \|00\bar{1}1\bar{1}1_6\rangle$					-1														-2	-1					-1		-1			-8		
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_1\rangle$						-1	1	1															-1	1		1						
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_2\rangle$				1		-10	-5																								1	-1
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_3\rangle$				-2										1																		
$\|0010\bar{1}\bar{1}\rangle\ \|00\bar{1}011_6\rangle$					-1																									-1		
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_1\rangle$						-1	1																									
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_2\rangle$				1		-10	-5																									
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_3\rangle$				-2										-2																	-2	-1
$\|01\bar{1}010\rangle\ \|0\bar{1}10\bar{1}0_6\rangle$	-2				-1				-2	-3	-3							-1	-4	-2					-2	-2	-1			-8		
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_1\rangle$						-1	1					-1	1										-1	1								
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_2\rangle$				1		-10	-5																			1					1	-1
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_3\rangle$				-2						1	-1			1				-1								-2						
$\|01\bar{1}1\bar{1}0\rangle\ \|0\bar{1}1\bar{1}10_6\rangle$	-2				-1				1	-1	1								-2	-1					-1					-1		
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_1\rangle$		1	-1																													
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_2\rangle$																																
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_3\rangle$				1	-1									-2																-1		
$\|1\bar{1}0010\rangle\ \|\bar{1}100\bar{1}0_6\rangle$	1								-2	-3	-3							-1	1	-1					-1	1						
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_1\rangle$												-1	1		-1	1							-1	1							1	
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_2\rangle$																		-1								1						
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_3\rangle$										1	-1							2								-2						
$\|010\bar{1}00\rangle\ \|0\bar{1}0100_6\rangle$	-2								1	-1	1								-2	-1					-1							

continued on next page

continued from previous page

	(110000)																														
	$\|33\rangle$	$\|34\rangle$	$\|35\rangle$	$\|36\rangle$	$\|37\rangle$	$\|38\rangle$	$\|39\rangle$	$\|40\rangle$	$\|41\rangle$	$\|42\rangle$	$\|43\rangle$	$\|44\rangle$	$\|45\rangle$	$\|46\rangle$	$\|47\rangle$	$\|48\rangle$	$\|49\rangle$	$\|50\rangle$	$\|51\rangle$	$\|52\rangle$	$\|53\rangle$	$\|54\rangle$	$\|55\rangle$	$\|56\rangle$	$\|57\rangle$	$\|58\rangle$	$\|59\rangle$	$\|60\rangle$	$\|61\rangle$	$\|62\rangle$	$\|63\rangle$	$\|64\rangle$
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_1\rangle$		1	-1									-1	1								1	-1										
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_2\rangle$																																
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_3\rangle$				1	-1					1	-1			1				-1	1	-1					-1	1	-1			4	1	-1
$\|1\bar{1}01\bar{1}0\rangle$ $\|\bar{1}10\bar{1}10_6\rangle$	1								1	-1	1								1	2					-1							
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_1\rangle$		1	-1																													
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_2\rangle$																																
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_3\rangle$					2																									-1		
$\|\bar{1}00010\rangle$ $\|1000\bar{1}0_6\rangle$	1								-2	2	-2								1	-1					2							
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_1\rangle$												-1	1		-1	1					1	-1						1	-1			1
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_2\rangle$																		-1									-1				1	
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_3\rangle$										1	-1							2	1	-1					-1	1	2			2		
$\|1\bar{1}1\bar{1}00\rangle$ $\|\bar{1}1\bar{1}100_6\rangle$	1				-1				1	-1	1								1	2					-1					2		
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_1\rangle$		1	-1																		1	-1										
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_2\rangle$																																
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_3\rangle$					2														1	-1					2		-1			4		
$\|\bar{1}001\bar{1}0\rangle$ $\|100\bar{1}10_6\rangle$	1								1	3	3							-1	1	2					2	1						
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_1\rangle$															-1	1	-1	-1										1	-1			1
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_2\rangle$																											-1				1	
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_3\rangle$																			1	2					-1		2			2		
$\|10\bar{1}001\rangle$ $\|\bar{1}0100\bar{1}_6\rangle$					-1																									2		
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_1\rangle$																					1	-1						1	-1			
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_2\rangle$																											-1					1
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_3\rangle$																			1	-1					2		2			2	1	2
$\|\bar{1}01\bar{1}00\rangle$ $\|10\bar{1}100_6\rangle$	1			1	2				1	3	3			1				2	1	2					2	1				2		
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_1\rangle$																	2	2														
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_2\rangle$																											2	1	2			1
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_3\rangle$																					1	2										
$\|10000\bar{1}\rangle$ $\|\bar{1}00001_6\rangle$		1	2																													
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_1\rangle$																	-1											1	-1			
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_2\rangle$																		-1									-1					1
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_3\rangle$										1	2							2	1	2					2	1	2			2	1	2
$\|\bar{1}1\bar{1}001\rangle$ $\|1\bar{1}100\bar{1}_6\rangle$				1	2									1																2		
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_1\rangle$																	2															
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_2\rangle$															2	1		2									2	1	2		1	4
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_3\rangle$												2	1								1	2										
$\|\bar{1}1000\bar{1}\rangle$ $\|1\bar{1}0001_6\rangle$		1	2																													
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_1\rangle$																	-1															
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_2\rangle$																		-1														
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_3\rangle$										1	2							2								2						
$\|0\bar{1}0001\rangle$ $\|01000\bar{1}_6\rangle$				2		20	10							1																	1	2
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_1\rangle$																	2											1	2			
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_2\rangle$															2	1		2														1
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_3\rangle$												2	1										2	1		1					1	2
$\|0\bar{1}100\bar{1}\rangle$ $\|01\bar{1}001_6\rangle$				1		22	11																									
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_1\rangle$																					1	2						1	2			
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_2\rangle$												2	1		2	1																1
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_3\rangle$																							2	1							1	2
$\|00\bar{1}100\rangle$ $\|001\bar{1}00_6\rangle$				1		22	11	1																		1						
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_1\rangle$		1	2																		1	2										
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_2\rangle$												2	1																			
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_3\rangle$						2	1																2	1								
$\|000\bar{1}10\rangle$ $\|0001\bar{1}0_6\rangle$								1																		1						
$\|0000\bar{1}0\rangle$ $\|000010_1\rangle$		1	2																													
$\|0000\bar{1}0\rangle$ $\|000010_2\rangle$																																
$\|0000\bar{1}0\rangle$ $\|000010_3\rangle$						2	1																									
$\|0000\bar{1}0\rangle$ $\|000010_6\rangle$								1																								
	$\sqrt{18}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{30}$	$\sqrt{24}$	$\sqrt{2064}$	$\sqrt{534}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{54}$	$\sqrt{54}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{42}$	$\sqrt{48}$	$\sqrt{30}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{18}$	$\sqrt{18}$	$\sqrt{48}$	$\sqrt{42}$	$\sqrt{30}$	$\sqrt{24}$	$\sqrt{24}$	$\sqrt{228}$	6	$\sqrt{108}$

Table 31: **CG coefficients for the (000000) dominant weight states of the 650-dimensional (100010) irrep in the product (100000)⊗(200000).** $\vert n\rangle$ is an abbreviation for $\vert 000000_n\rangle$. Numbering of the degenerate states is consistent with table I in ref.[5]. Each CGC should be divided by the respective number in the last row to maintain $\langle n\vert n\rangle = 1$.

	(100010)																			
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$
$\vert 100000\rangle\ \vert \bar{1}00000_1\rangle$	4											5			10					
$\vert 100000\rangle\ \vert \bar{1}00000_2\rangle$	-3	5									10									
$\vert 100000\rangle\ \vert \bar{1}00000_3\rangle$	2						10		5											
$\vert 100000\rangle\ \vert \bar{1}00000_6\rangle$	-1			5															5	10
$\vert \bar{1}10000\rangle\ \vert 1\bar{1}0000_1\rangle$	4		4									5	5		-5					
$\vert \bar{1}10000\rangle\ \vert 1\bar{1}0000_2\rangle$	-3	5	-3		5						-5									
$\vert \bar{1}10000\rangle\ \vert 1\bar{1}0000_3\rangle$	2		2	5			-5	5	5											
$\vert \bar{1}10000\rangle\ \vert 1\bar{1}0000_6\rangle$	-1		-1	-5														5	5	-5
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_1\rangle$			4			4							5	5	5					
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_2\rangle$			-3		5	-3		5		5	5									
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_3\rangle$			2	5		2	5	-5												
$\vert 0\bar{1}1000\rangle\ \vert 01\bar{1}000_6\rangle$			-1	-5		-1	-5		-5									-5	-10	-5
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_1\rangle$						4				5				5	-5	4	5			
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_2\rangle$						-3		5		-5	-5					-3		5		
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_3\rangle$				-5	-5	2	5	-5			5					2		-5		5
$\vert 00\bar{1}101\rangle\ \vert 001\bar{1}0\bar{1}_6\rangle$		-5				-1	-5		-5		-5					-1		5	5	
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_1\rangle$										5					5	4	-1			
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_2\rangle$							-5	-5		-5	-5			-5	-5	-3	-3	-5		
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_3\rangle$					-5						5		-5		5	2	2	5		-5
$\vert 000\bar{1}11\rangle\ \vert 0001\bar{1}\bar{1}_6\rangle$		-5									-5	-5			-5	-1	-1	-5	-5	
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_1\rangle$										-1					5	4	5			
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_2\rangle$										2	5					-3		5		
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_3\rangle$				10	5					2	-5					2		-5		5
$\vert 00010\bar{1}\rangle\ \vert 000\bar{1}01_6\rangle$		5					5	5	10	-1	5					-1		5	5	
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_1\rangle$										-5	5				5	-5	-1	-1		
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_2\rangle$						-5	5							-5	-5		-3	2		
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_3\rangle$			-5										-5		5		2	-3		5
$\vert 0000\bar{1}1\rangle\ \vert 00001\bar{1}_6\rangle$	-5											-5			-5		-1	4	5	
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_1\rangle$						4				-1				-1	-5	1	-1			
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_2\rangle$						-3	5			2	5			2	5	-3	-3	-5		
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_3\rangle$				-5	5	2	-5	5		2	-5		5	2	-5	-2	2	5		-5
$\vert 001\bar{1}1\bar{1}\rangle\ \vert 00\bar{1}1\bar{1}1_6\rangle$		5				-1		-5	-5	-1	5	5		-1	5	-1	-1	-5	-5	
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_1\rangle$						-5		-1			-5			-1	-5	-5	-1	-1		
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_2\rangle$						5	-5	2						2	5		-3	2		
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_3\rangle$			5	5			5	-3					5	2	-5		2	-3		5
$\vert 0010\bar{1}\bar{1}\rangle\ \vert 00\bar{1}011_6\rangle$	5							4	5			5		-1	5		-1	4	5	
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_1\rangle$			4			4							-1	-1	5					
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_2\rangle$			-3			-3	5						2	2	-5		5	5		5
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_3\rangle$			2	5		2	-5	5					-3	2	5					
$\vert 01\bar{1}010\rangle\ \vert 0\bar{1}10\bar{1}0_6\rangle$		-10	-1	-5	-5	-1		-5	-5		-5	-5	-1	-1	-5					
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_1\rangle$			-5		-1	-5		-1		-1	5		-1	-1	5					
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_2\rangle$			5		2	5	-5	2		2	-5		2	2	-5	5	5			-5
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_3\rangle$			-5	-5	-3		5	-3		2	5		-3	2	5					
$\vert 01\bar{1}1\bar{1}0\rangle\ \vert 0\bar{1}1\bar{1}10_6\rangle$	-5	5		5	4			4	5	-1		-5	-1	-1	-5					
$\vert 1\bar{1}0010\rangle\ \vert \bar{1}100\bar{1}0_1\rangle$	4		4									-1	-1		-5					
$\vert 1\bar{1}0010\rangle\ \vert \bar{1}100\bar{1}0_2\rangle$	-3		-3									2	2		5		-5	-5	5	5
$\vert 1\bar{1}0010\rangle\ \vert \bar{1}100\bar{1}0_3\rangle$	2		2	5			5	-5	5			-3	-3	-5	-5					
$\vert 1\bar{1}0010\rangle\ \vert \bar{1}100\bar{1}0_6\rangle$	-1	5	-1	-5	-5					-5	5	4	-1		5					
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_1\rangle$			5	-1	-1	5	5			-1	5									
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_2\rangle$			-5	2	2					2	-5					5				5
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_3\rangle$			5	2	-3					2	5									
$\vert 010\bar{1}00\rangle\ \vert 0\bar{1}0100_6\rangle$	5	5		-1	4					-1		10	5		5					

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continued from previous page

	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$	$\vert 7\rangle$	$\vert 8\rangle$	$\vert 9\rangle$	$\vert 10\rangle$	$\vert 11\rangle$	$\vert 12\rangle$	$\vert 13\rangle$	$\vert 14\rangle$	$\vert 15\rangle$	$\vert 16\rangle$	$\vert 17\rangle$	$\vert 18\rangle$	$\vert 19\rangle$	$\vert 20\rangle$
	(100010)																			
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_1\rangle$	-5	-1	-5		-1						-5	-1	-1		-5					
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_2\rangle$	5	2	5		2						5	2	2		5	-5	-5		-5	-5
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_3\rangle$	-5	-3	-5	-5	-3	-5	-5		-5	-5	-5	-3	-3	-5	-5					
$\vert 1\bar{1}01\bar{1}0\rangle\ \vert\bar{1}10\bar{1}10_6\rangle$	5	-1		5	4					5		4	-1		5					
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_1\rangle$	4											-1			4					
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_2\rangle$	-3											2			-3		5	10	5	
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_3\rangle$	2							10	5			-3		5	2					
$\vert\bar{1}00010\rangle\ \vert 1000\bar{1}0_6\rangle$	-1	5		5	10					5		4	5		-1					
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_1\rangle$	5	-1	5	-1	-1		-5	-1	-1		-5									
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_2\rangle$	-5	2	-5	2	2	-5	5	2	2		5					-5			5	5
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_3\rangle$	5	-3	5	2	-3	5		-3	2	-5	-5									
$\vert 1\bar{1}1\bar{1}00\rangle\ \vert\bar{1}1\bar{1}100_6\rangle$	-5	-1		-1	4			4	-1	5		-5	5	5	-5					
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_1\rangle$	-5	-1									4	-1			4					
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_2\rangle$	5	2									-3	2			-3	5	5	-5	-5	
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_3\rangle$	-5	-3				5		-5	-5	5	2	-3		5	2					
$\vert\bar{1}001\bar{1}0\rangle\ \vert 100\bar{1}10_6\rangle$	5	-1	5	-5	-5					-5	-1	4	5		-1					
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_1\rangle$	-5		-5				-5	-1	-1									-1	-1	-5
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_2\rangle$	5					-5	5	2	2									2	-3	
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_3\rangle$	-5	5				5		-3	2	5	5					5		-3	2	
$\vert 10\bar{1}001\rangle\ \vert\bar{1}0100\bar{1}_6\rangle$	5							4	-1			5		5	5		5	4	-1	
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_1\rangle$	5	-1					4		-1		4									
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_2\rangle$	-5	2				5	-3		2		-3					5		5	5	
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_3\rangle$	5	-3				-5	2	5	2	5	2									
$\vert\bar{1}01\bar{1}00\rangle\ \vert 10\bar{1}100_6\rangle$	-5	-1	-5		-5		-1	-5	-1	-5	-1	-5	-10	-5						
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_1\rangle$	5		5	-5														-1	-1	-5
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_2\rangle$	-5					5	-5		-5									2	-3	
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_3\rangle$	5	-5									-5					5		-3	2	
$\vert 10000\bar{1}\rangle\ \vert\bar{1}00001_6\rangle$	-5											-5			-5		5	4	-1	
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_1\rangle$	-5			-1			4		-1										-1	4
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_2\rangle$	5		5	2		5	-3		2									-5	-3	-3
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_3\rangle$	-5	5	-5	2	5	-5	2	5	2	-5						-5		5	2	2
$\vert\bar{1}1\bar{1}001\rangle\ \vert 1\bar{1}100\bar{1}_6\rangle$	5		5	-1			-1	-5	-1			5	5	-5			-5	-5	-1	-1
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_1\rangle$	5			5															-1	4
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_2\rangle$	-5		-5	-5		-5		-5	-5									-5	-3	-3
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_3\rangle$	5	-5	5		-5											-5		5	2	2
$\vert\bar{1}1000\bar{1}\rangle\ \vert 1\bar{1}0001_6\rangle$	-5		-5									-5	-5				-5	-5	-1	-1
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_1\rangle$	5			-1															5	4
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_2\rangle$			5	2														5		-3
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_3\rangle$			-5	2	5					10						5		-5		2
$\vert 0\bar{1}0001\rangle\ \vert 01000\bar{1}_6\rangle$			5	-1		5		5					5	10			5	5		-1
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_1\rangle$	-5			5			4		5										5	4
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_2\rangle$			-5	-5			-3	5										5		-3
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_3\rangle$			5		-5	5	2	-5		-5						5		-5		2
$\vert 0\bar{1}100\bar{1}\rangle\ \vert 01\bar{1}001_6\rangle$			-5			-5	-1						-5	-5			5	5		-1
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_1\rangle$	5	5					4		5		4									
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_2\rangle$			5	5	5		-3	5			-3									
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_3\rangle$						5	2	-5		5	2									
$\vert 00\bar{1}100\rangle\ \vert 001\bar{1}00_6\rangle$						-5	-1			-5	-1			-5		-5	-10	-5		
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_1\rangle$	-5	5									4	5			4					
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_2\rangle$			-5		5						-3		5		-3					
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_3\rangle$						-5		5		5	2			5	2					
$\vert 000\bar{1}10\rangle\ \vert 0001\bar{1}0_6\rangle$										-5	-1				-1	-5	5	5		
$\vert 0000\bar{1}0\rangle\ \vert 000010_1\rangle$	10											5			4					
$\vert 0000\bar{1}0\rangle\ \vert 000010_2\rangle$			10										5		-3					
$\vert 0000\bar{1}0\rangle\ \vert 000010_3\rangle$						10								5	2					
$\vert 0000\bar{1}0\rangle\ \vert 000010_6\rangle$										5					-1	10	5			
	$\sqrt{540}$ each state																			